this case weakly causal systems can be used. We will not go into this any further and we refer to [4] for the details.

The estimator (4) which has been built based on the matrix $K(s)$, constructed above, can be implemented as a Roesser model again and then we have obtained an estimator for R_{kh} in the model (1).

Because

$$S_{k,h+1} = A_3 R_{kh} + A_4 S_{kh} + B_2 u_{kh} + G_2 v_{kh}$$

we can also construct an estimator for S_{kh}.

$$\hat{S}_{k,h+1} = A_3 \hat{R}_{kh} + A_4 \hat{S}_{kh} + B_2 u_{kh} + L\left(y - C_1 \hat{R}_{kh} - C_2 \hat{S}_{kh}\right).$$

The dynamics of the associated error equation may be improved by choosing L such that $A_4 - LC_2$ has a "better" spectrum than A_4.

Combining the error equation (5) and the error equation associated with $(\hat{S}_{kh} - S_{kh})$ it is not difficult to prove that the resulting error equation for $(\hat{R}_{kh} - R_{kh})$ and $(\hat{S}_{kh} - S_{kh})$ is stable. Of course one has to choose $K(s)$, in $A(s) - K(s)$, based on some criterion (which may be related to the covariance of $(\hat{R}_k(s) - R_k(s))$. Also for the selection of L, in $A_4 - LC_2$, one has to use some criterion.

One way to do this (suboptimally) is by parameterizing $K(s)$ and then one might select a satisfactory one by means of an optimization technique. A special case is where $K(s)$ is chosen *a priori* such that $\det[zI - A(s) + K(s)C(s)]$ is a polynomial independent of s. Then all such $K(s)$ can be parameterized by the eigenvalues $\lambda_1, \cdots, \lambda_n$ of $A(s) - K(s)C(s)$. In this case the error equation (5) is a separable 2-D system. This means that a Roesser model for this system can be chosen such that the corresponding A_3-matrix or A_2-matrix is zero. This property may be advantageous with respect to the computation of covariance matrices for $\hat{R}_{kh} - R_{kh}$ as can be seen in [1] (the structure of the equation is considerably simplified). This is because the error (in this case) is a sum of independent random variables as can easily be seen.

CONCLUSION

A new method of construction of 2-D filters has been described. The filter is given as a local state estimator for a Roesser model. The actual construction of the filter is based on the design of an observer for the global state in a state-space model (column to column propagation) which is equivalent to the (local state space) Roesser model.

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Multiport Approach to Multiple-Fault Location in Analog Circuits

J. A. STARZYK AND J. W. BANDLER

***Abstract*—This paper deals with the multiport method for multiple-fault location in linear analog circuits. A hybrid multiport description of the linear network has been used in the presentation, which generalizes and explains proposals made by Biernacki and Bandler. The problem of consistency of the chosen set of equations used for fault identification is discussed. The restrictions of the method are explained on the basis of network topology.**

I. INTRODUCTION

Multiple-fault location is an important problem in testing analog circuits, particularly when the number of measurements is too small to evaluate all of the network elements. Various researchers have discussed and elaborated upon the problem of fault location [1]–[7] under the assumption that the network elements assume their nominal values (or are close to them) and only a few elements are faulty.

In a recent paper by Biernacki and Bandler [3], the multiport approach to fault location was discussed and some necessary conditions on the set of equations used for fault identification were formulated. The concept of block dependency of these equations is of special importance in the identification problem and is discussed in particular in this paper for a general hybrid description of the multiport used to relate ports of fault to ports of measurement. Certain conjectures made by Biernacki and Bandler are addressed in a rigorous manner.

II. HYBRID REPRESENTATION FOR MULTIPORT APPROACH TO FAULT LOCATION

Assume for simplicity that the linear network under investigation contains one-port elements and controlled sources only. Assume also that the network has $n+1$ nodes, e elements with f of them faulty. To identify all the faults we measure m voltages in the network, $m > f$.

Changes in element values w.r.t. the nominal can be represented by current sources in parallel with elements (for one ports and controlled current sources) or by voltage sources in series with elements (for one ports and controlled voltage sources). See Fig. 1. Changes w.r.t. nominal values, i.e., the *faults* can be represented as loads of the $(m+f)$-port network consisting of all the elements of the original network which are at their nominal values. See Fig. 2. Assuming that the hybrid matrix $\boldsymbol{H}$ of the $(m+f)$-port exists we obtain the relation

$$\begin{bmatrix} \boldsymbol{V}^M \\ \boldsymbol{V}^{F_1} \\ \boldsymbol{I}^{F_2} \end{bmatrix} = \boldsymbol{H} \begin{bmatrix} \boldsymbol{I}^M \\ \boldsymbol{I}^{F_1} \\ \boldsymbol{V}^{F_2} \end{bmatrix} \tag{1}$$

where

$$\boldsymbol{V}^M \triangleq \left[V_1^M \cdots V_m^M\right]^T \qquad \boldsymbol{I}^M \triangleq \left[I_1^M \cdots I_m^M\right]^T \tag{2}$$

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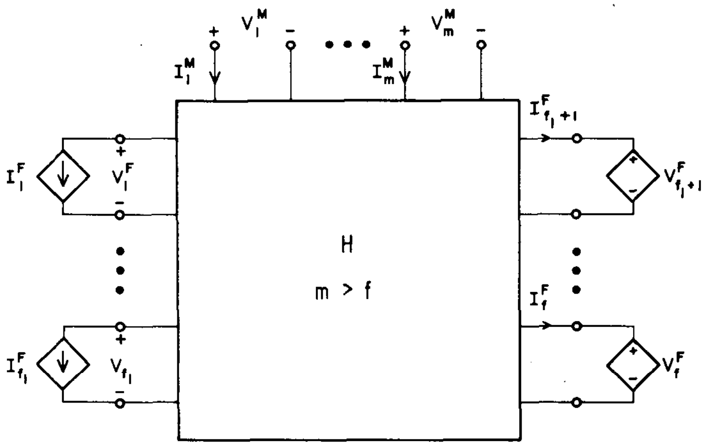

Fig. 1. Representation for changes in element values. (a) D_1 denotes a one-port or controlled current source. (b) D_2 denotes a one-port or controlled voltage source, ν denotes the controlling voltage or current.

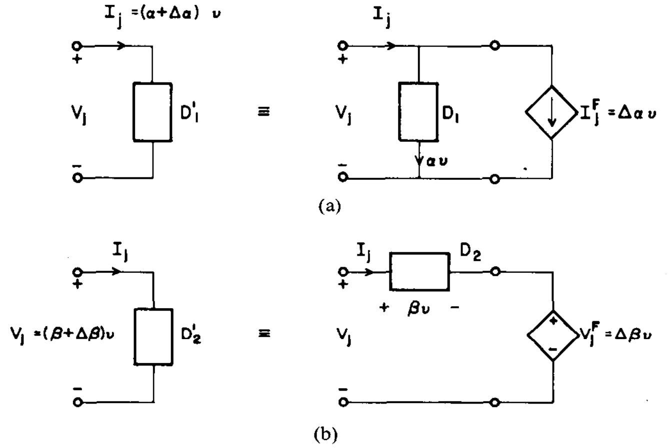

Fig. 2. Network with faults represented as controlled sources.

are measured voltages and currents,

$$V^{F_1} \triangleq \left[V_1^F \cdots V_{f_1}^F\right]^T \qquad V^{F_2} \triangleq \left[V_{f_1+1}^F \cdots V_f^F\right]^T \tag{3}$$

are voltages at fault ports, and

$$I^{F_1} \triangleq \left[I_1^F \cdots I_{f_1}^F\right]^T \qquad I^{F_2} \triangleq \left[I_{f_1+1}^F \cdots I_f^F\right]^T \tag{4}$$

are currents flowing through fault ports.

Equation (1) can be represented in the simpler form

$$\begin{bmatrix} V^M \\ R^F \end{bmatrix} = \begin{bmatrix} H_{MM} & H_{MF} \\ H_{FM} & H_{FF} \end{bmatrix} \begin{bmatrix} I^M \\ S^F \end{bmatrix} \tag{5}$$

where

$$S^F = \begin{bmatrix} I^{F_1} \\ V^{F_2} \end{bmatrix} \qquad R^F = \begin{bmatrix} V^{F_1} \\ I^{F_2} \end{bmatrix} \tag{6}$$

are source and response vectors at fault ports. When there are no faults in the network we obtain the nominal response vector

$$\begin{bmatrix} V^{M0} \\ R^{F0} \end{bmatrix} = H \begin{bmatrix} I^M \\ 0 \end{bmatrix}. \tag{7}$$

Hence, the voltage change vector $\Delta V^M \triangleq V^M - V^{M0}$ can be expressed as

$$\Delta V^M = H_{MF} S^F. \tag{8}$$

Assuming that H_{MF} is of full column rank, the necessary condition for the set F of network elements to contain all the faults, is given by the relation

$$\left(\bar{H}_{MF} - 1\right)\Delta V^M = 0 \tag{9}$$

where

$$\bar{H}_{MF} \triangleq H_{MF}\left(H_{MF}^T H_{MF}\right)^{-1} H_{MF}^T. \tag{10}$$

H_{MF} is called the test matrix. Equation (9) allows us to check whether the assumed set F contains all faults existing in the network on the basis of the measured voltage change vector ΔV^M.

The test matrix H_{MF} can be designed using the adjoint network analysis. For the adjoint network with $\hat{S}^F = 0$ we obtain (cf. [8])

$$\begin{bmatrix} \hat{V}^{F_1} \\ -\hat{I}^{F_2} \end{bmatrix} = H_{MF}^T \hat{I}^M. \tag{11}$$

For m linearly independent excitations $\hat{I}^{M1}, \hat{I}^{M2}, \cdots, \hat{I}^{Mm}$ we have

$$\begin{bmatrix} \hat{V}^{F_1 1} & \hat{V}^{F_1 2} & \cdots & \hat{V}^{F_1 m} \\ -\hat{I}^{F_2 1} & -\hat{I}^{F_2 2} & \cdots & -\hat{I}^{F_2 m} \end{bmatrix} = H_{MF}^T \begin{bmatrix} \hat{I}^{M1} & \hat{I}^{M2} & \cdots & \hat{I}^{Mm} \end{bmatrix} \tag{12}$$

and

$$\begin{bmatrix} \hat{I}^{M1} & \hat{I}^{M2} & \cdots & \hat{I}^{Mm} \end{bmatrix} = 1 \tag{13}$$

giving the test matrix

$$H_{MF}^T = \begin{bmatrix} \hat{V}^{F_1 1} & \cdots & \hat{V}^{F_1 m} \\ -\hat{I}^{F_2 1} & \cdots & -\hat{I}^{F_2 m} \end{bmatrix}. \tag{14}$$

Having calculated voltages and currents in all adjoint network elements with different current excitations, $\hat{I}^{Mi}$, we can obtain the test matrix H_{MF} corresponding to any set F of faulty elements. Thus the adjoint network analysis does not have to be repeated if the set of predicted faulty elements is changed.

III. THE BLOCK DEPENDENCY PROBLEM

Fulfilling the relation (9) is a necessary but not sufficient condition on the set F to contain all the faults in the network. One of the important reasons is the concept of block dependency of two overdetermined systems of equations. In this section we will specifically address this problem.

Systems

$$A_1 x_1 = b \quad \text{and} \quad A_2 x_2 = b \tag{15}$$

are *block dependent* if for any $\boldsymbol{b}$ both are consistent or both are inconsistent. It was shown [3] that two systems are block dependent if and only if

$$\bar{A}_1 A_2 = A_2 \quad \text{or} \quad \bar{A}_2 A_1 = A_1 \tag{16}$$

where

$$\bar{A} \triangleq A(A^T A)^{-1} A^T. \tag{17}$$

The reasons for block dependency are very often connected with the particular form of the test matrix $\boldsymbol{H}_{MF}$. We will discuss them first.

Lemma 1

Let us assume that we have the overdetermined system of equations

$$\boldsymbol{A}\boldsymbol{x} = \boldsymbol{b} \tag{18}$$

where $\boldsymbol{A} = \begin{bmatrix} \boldsymbol{a}_1 & \cdots & \boldsymbol{a}_f \end{bmatrix}$ is an $(m \times f)$ matrix, $f < m$, and rank $\boldsymbol{A} = f$. Every overdetermined system of equations

$$\boldsymbol{B}\boldsymbol{x}_2 = \boldsymbol{b} \tag{19}$$

where $\boldsymbol{B} = \begin{bmatrix} \boldsymbol{b}_1 & \cdots & \boldsymbol{b}_f \end{bmatrix}$, rank $\boldsymbol{B} = f$ and $\boldsymbol{b}_i$, $i = 1, \cdots, f$ are linear combinations of $\boldsymbol{a}_i \in \boldsymbol{A}$, is block dependent to the system (18), so according to [3] we can write $\boldsymbol{A} \sim \boldsymbol{B}$.

Proof: Assume that $\boldsymbol{b}_i = \boldsymbol{a}_i$, $i = 1, \cdots, f-1$, and $\boldsymbol{b}_f = \boldsymbol{a}_f + \Sigma_j k_j \boldsymbol{a}_j$, where k_j are scalars. We prove first the following equivalence:

$$\bar{\boldsymbol{A}}\boldsymbol{A}_f = \boldsymbol{A}_f \tag{20}$$

where

$$\boldsymbol{A}_f = \sum_j k_j \boldsymbol{a}_j \tag{21}$$

and $\boldsymbol{A}$ is as in (18). We have $\bar{\boldsymbol{A}}\boldsymbol{A} = \boldsymbol{A}$, so

$$\begin{aligned} \bar{\boldsymbol{A}}\boldsymbol{A}_f &= \bar{\boldsymbol{A}} k_1 \boldsymbol{a}_1 + \bar{\boldsymbol{A}} k_2 \boldsymbol{a}_2 + \cdots + \bar{\boldsymbol{A}} k_f \boldsymbol{a}_f \\ &= k_1 \boldsymbol{a}_1 + k_2 \boldsymbol{a}_2 + \cdots + k_f \boldsymbol{a}_f = \boldsymbol{A}_f. \end{aligned}$$

Now we can easily check that

$$\begin{aligned} \bar{\boldsymbol{A}}\boldsymbol{B} &= \bar{\boldsymbol{A}}\begin{bmatrix} \boldsymbol{a}_1 & \boldsymbol{a}_2 & \cdots & \boldsymbol{a}_f + \boldsymbol{A}_f \end{bmatrix} \\ &= \begin{bmatrix} \boldsymbol{a}_1 & \boldsymbol{a}_2 & \cdots & \boldsymbol{a}_f + \boldsymbol{A}_f \end{bmatrix} = \boldsymbol{B}. \end{aligned} \tag{22}$$

We can extend the proof to the case when every column of $\boldsymbol{B}$ is a linear combination of columns of the $\boldsymbol{A}$ matrix when rank $\boldsymbol{B} = f$.

Now we can formulate the following important result.

Result 1

If the set of faulty elements contains a subset consisting of either

(a) a circuit formed by one-ports, controlled current sources or currents that control faulty voltage or current sources;

(b) a cutset formed by one-ports, controlled voltage sources or voltages that control faulty voltage or current sources;

then the test matrix $\boldsymbol{H}_{MF}$ of (8) contains linearly dependent columns.

Proof: The proof of Result 1 is connected with the adjoint network interpretation of the test matrix as described in (11). The jth column of the $\boldsymbol{H}_{MF}$ matrix ($j \leqslant f_1$) corresponds to voltages on the element e_j calculated for all independent current excitations and nominal values of elements. So if faulty elements (and/or controlled voltages) form a circuit then the voltages are dependent. The same dependency holds for all excitations and we find columns of $\boldsymbol{H}_{MF}$ linearly dependent. A similar argument can be used for the cutset formed by faulty elements. The jth column of the $\boldsymbol{H}_{MF}$ matrix ($j > f_1$) corresponds to currents flowing through the element e_j calculated for all independent excitations. Thus if faulty elements (and/or controlled currents) form a cutset then the currents are dependent. This leads to the linear dependency of the columns of $\boldsymbol{H}_{MF}$.

A corollary follows immediately from Result 1.

Corollary 1

If the set of faulty elements contains any subset defined in Result 1 then the multiport method cannot be used for fault location.

Although Corollary 1 is of a negative nature it provides precise information about the topological restriction on the multiport method, thus being a constructive extension of Theorem 2 given by Biernacki and Bandler [2].

From Lemma 1 and the proof of Result 1 it is clear that even if one of the elements from the subsets defined in Result 1 (i.e., from a circuit or a cutset) is not faulty the multiport method will not provide unique results. Assume that the set of faulty elements in the network $F = \{e_1, e_2, \cdots, e_f\}$, and that elements $e_{l+1}, e_{l+2}, \cdots, e_f$ together with e_{f+1} form a circuit or cutset (e_{f+1} may be a controlled variable as well as an element). Under this assumption we can prove the following lemma.

Lemma 2

If $\boldsymbol{H}_{MF_i}$ denotes the test matrix constructed for the set F_i of predicted faulty elements then

$$\boldsymbol{H}_{MF_1} \sim \boldsymbol{H}_{MF_2} \sim \cdots \sim \boldsymbol{H}_{MF_{f-l+1}} \tag{23}$$

where

$$\begin{aligned} F_1 = \{e_1, \cdots, e_f\} \qquad F_2 = \{e_1, \cdots, e_{f-1}, e_{f+1}\} \\ F_{f-l+1} = \{e_1, \cdots, e_l, e_{l+2}, \cdots, e_{f+1}\}. \end{aligned} \tag{24}$$

Proof: The column of $\boldsymbol{H}_{MF_i}$ ($i > 1$) matrix which corresponds to the element e_{f+1} is a linear combination of columns which correspond to the elements $e_{l+1}, \cdots, e_f$ (compare with the proof of Result 1). So, according to Lemma 1, we have the relation (23).

This case occurred in Example 2 of Biernacki and Bandler [3], where the matrices $\boldsymbol{Z}_{mx}^{13}$, $\boldsymbol{Z}_{mx}^{23}$, and $\boldsymbol{Z}_{mx}^{12}$ were block dependent.

Not only does linear dependency of columns influence the possibility of multiple fault location, but linear dependency of rows does as well. It is evident that the number of independent voltage measurements should at least be equal to the number of columns of the test matrix to obtain full column rank. The following lemma is helpful to a better understanding of the influence of row dependency on the solvability of the fault location problem.

Lemma 3

If the number of *independent* adjoint current excitations is less than or equal to the number of columns of the test matrix we cannot locate the faults using the multiport approach.

Proof: Assume for simplicity that we have the overdetermined system of equations

$$\boldsymbol{H}_{MF}\boldsymbol{S}^F = \Delta \boldsymbol{V}^M \tag{25}$$

where $\boldsymbol{H}_{MF}$ is an $(f+1) \times f$ matrix with rank $\boldsymbol{H}_{MF} = f$. Then $\boldsymbol{H}_{MF}$ can be presented in the form

$$\boldsymbol{H}_{MF} = \begin{bmatrix} \boldsymbol{a}_1^T \\ \vdots \\ \boldsymbol{a}_f^T \\ \sum_{j=1}^{f} k_j \boldsymbol{a}_j^T \end{bmatrix}. \tag{26}$$

Condition (9) is then equivalent to

$$\Delta V^M_{f+1} = \sum_{j=1}^{f} k_j \Delta V^M_j \qquad (27)$$

where

$$\Delta V^M = \begin{bmatrix} \Delta V^M_1 \\ \vdots \\ \Delta V^M_{f+1} \end{bmatrix}. \qquad (28)$$

But relation (27) is always fulfilled if the current excitations are linearly dependent, because each row of the $\boldsymbol{H}_{MF}$ matrix corresponds to voltages or currents on elements $e_1, \cdots, e_f$ calculated for one adjoint current excitation and nominal values of the elements (cf. (14)). So if the current in, say, the $(f+1)$th excitation is a linear combination of the first f excitations then the $(f+1)$th row of the $\boldsymbol{H}_{MF}$ matrix is the same combination of the first f rows. According to the structure of $\boldsymbol{H}_{MF}$ we can see that the same combination of measurement voltages will appear in the $(f+1)$th measurement, because excitations in the adjoint network are imposed at the same ports as measurements in the faulty network (see (12) and (8)). Thus condition (27) is fulfilled independently from $e_1, e_2, \cdots, e_f$.

Corollary 2

The maximum number of faults which can be located by the multiport method is equal to the number of nodes in the network minus two.

This corollary is the simple result of Lemma 3 because the maximum number of independent excitations is equal to number of nodes minus one.

IV. CONCLUSIONS

The hybrid multiport approach to fault location has been presented together with a detailed and rigorous discussion of the implications of block dependency. Topological restrictions on the method have been derived from the analysis. The theory is applicable to multiple-fault location in linear networks due to current excitations at a single frequency. We feel that the present paper permits a deeper understanding of the limitations of using the multiport method for fault location. The influence of tolerances can, in practice, be handled for this approach in a similar manner to the treatment of Biernacki and Bandler [3].

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A New Look at the Magill Adaptive Filter as a Practical Means of Multiple Hypothesis Testing

R. GROVER BROWN

***Abstract*—The Magill adaptive filter can be used to detect known signals in the presence of Gauss–Markov noise. In this application, the various hypotheses are accounted for outside the bank of Kalman filters, and thus all filters have the same gains and error covariances. This commonality makes it feasible to use the Magill scheme in large-scale multiple-hypothesis testing applications.**

I. INTRODUCTION

It has been over 20 years since Kalman published his recursive solution of the least squares filtering problem [1]. In addition to its theoretical elegance, discrete Kalman filtering has proved to be eminently practical. No doubt this accounts for its continued durability. About five years after Kalman's paper, Magill published a paper on adaptive recursive estimation [2]. In Magill's estimator, the unknown parameter was assumed to be a discrete random variable with a finite number of possible realizations. His solution then called for a bank of parallel Kalman filters, all operating on the measurement sequence simultaneously. The outputs of the filters were then blended together to yield the optimal estimate.[1] Magill's adaptive filter is often mentioned in discussions of adaptive filtering [3], [4], but it is sometimes passed off as being impractical because of the discretization of the unknown parameter and the bank of parallel Kalman filters. Certainly the discretization should be no cause for discarding Magill's scheme, because many practical hypothesis testing problems are inherently discrete. Furthermore, it will be shown that a high degree of commonality can be achieved in the bank of filters when detecting known signals in the presence of Gauss–Markov noise. In this case, it is feasible to use the Magill scheme as a practical means of solving large-scale multiple hypothesis testing problems.

II. THE DISCRETE KALMAN FILTER

The discrete Kalman filter is well known and tutorial discussions are readily available in various textbooks [3], [4]. Thus only a summary of the key equations will be given here for convenient reference. The vector process model and measurement relationship are assumed to be of the form:

Process Model:

$$\boldsymbol{x}_{k+1} = \boldsymbol{\phi}_k \boldsymbol{x}_k + \boldsymbol{w}_k, \quad E\left[\boldsymbol{w}_j \boldsymbol{w}_k^T\right] = \begin{cases} \boldsymbol{Q}_k, & j = k \\ 0, & j \neq k. \end{cases} \qquad (1)$$

Measurement Equation:

$$\boldsymbol{z}_k = \boldsymbol{H}_k \boldsymbol{x}_k + \boldsymbol{v}_k, \quad E\left[\boldsymbol{v}_j \boldsymbol{v}_k^T\right] = \begin{cases} \boldsymbol{R}_k, & j = k \\ 0, & j \neq k. \end{cases} \qquad (2)$$

In the usual Kalman filter the $\boldsymbol{\phi}_k$, $\boldsymbol{Q}_k$, $\boldsymbol{H}_k$, and $\boldsymbol{R}_k$ parameters are assumed to be known. The recursive procedure for obtaining the optimal sequence of estimates may now be stated as follows:

(1) Enter the recursive loop with an initial *a priori* estimate $\hat{\boldsymbol{x}}_k^-$ and its error covariance $\boldsymbol{P}_k^-$. (Usually we let the first measurement occur at $k = 0$.)

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[1] Magill's parallel processor is also known as a multiple model estimation algorithm (MMEA).

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