A generalized fault diagnosis in dynamic analog circuits

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SUMMARY

Fault diagnosis of analog circuits is essential for analog and mixed-signal systems testing and maintenance. A new method is proposed in this paper for multiple fault diagnosis of linear analog circuits in frequency domain. Woodbury formula is applied to the modified nodal equation to construct the fault diagnosis equation, which relates the limited measured circuit responses with the multiple faults inside the circuit in a linear way. A recently developed ambiguity group locating technique is modified here to identify the faulty parameters directly. Computation cost is reduced compared to combinatorial search in traditional fault verification methods. Only one node is needed for voltage measurement, but multiple excitations and corresponding measurements on accessible nodes are required for fault identification. Parameter evaluation can provide the exact solution to the deviated values of faulty parameters. The faulty parameter deviations can have any finite values. Example circuits are provided to illustrate the proposed method. Two other methods for multiple analog fault diagnosis sharing the same mechanism as the method proposed in this paper are also briefly described. The proposed method is extremely effective for the circuit with very limited accessible nodes and is also computationally efficient.

KEY WORDS: fault diagnosis; fault verification; linear analog circuits; ambiguity groups

1. INTRODUCTION

Fault diagnosis of analog circuits has been one of the most challenging topics for researchers and test engineers since the 1970s. Given the circuit topology and nominal circuit parameter values, fault diagnosis is to obtain the exact information about the faulty circuit based on the analysis of the limited measured circuit responses. There are three dominant and distinct stages in the process of fault diagnosis: fault detection to find out if the circuit under test (CUT) is faulty comparing with the fault-free circuit, or gold circuit (This stage is usually called test in industry), fault identification to locate where the faulty parameters are inside the faulty circuit, and parameter evaluation to obtain how much the faulty parameters deviated from their nominal values and to obtain values of other circuit parameters such as branch and nodal voltages. The bottlenecks of analog fault diagnosis primarily lie in the inherited features of analog circuits: nonlinearity, parameter tolerances, limited accessible nodes, and lack of efficient models. Multiple fault diagnosis techniques are even less developed than single fault diagnosis because it is more

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difficult to model and detect multiple faults, particularly in the presence of tolerance or measurement noise. In addition, in multiple fault situation one fault's effect on the circuit could be masked by the effects of the other faults. Generally speaking, there is no widely accepted paradigm for analog test or fault diagnosis even with the introduction of IEEE 1149.4 standard for mixed-signal test bus.

With recent sharp development of electronic design automation tools and widespread application of analog VLSI chips and mixed-signal systems in the area of wireless communication, networking, neural network and real-time control, the interests in analog test and fault diagnosis revives. System-on-chip solutions favored by modern electronics pose new challenges in this topic such as increased complexity and reduced die size and accessibility. Several good periodical reviews on this topic appeared in 1979 [1], 1985 [2], 1991 [3] and 1998 [4], respectively. The papers [5-8] are examples of research efforts after 1998.

In [9], a method was proposed for single fault diagnosis in linear analog circuit. Multiple excitations are required and Woodbury formula in matrix theory is applied to locate the faulty parameter. This method is also applied to multiple fault diagnosis by decomposition technique assuming that each sub-circuit contains at most a single faulty parameter. In this paper, the method developed in [9] is generalized and extended to multiple fault diagnosis of linear analog circuits in frequency domain. In our work, multiple excitations and Woodbury formula are also required for fault identification. However, a recently developed ambiguity group locating technique is applied for fault identification which reduces computational cost of the test method. Multiple faults can be located directly and efficiently, thus eliminating the requirement for decomposition and the corresponding restrictions. Moreover, the methodology developed in our work, (i.e., constructing fault diagnosis equation on the basis of the analysis of the fault-free circuit and the measured responses of faulty circuit, then applying the ambiguity group locating technique to identify the faulty parameters, finally evaluating all parameter values of faulty circuit exactly,) can be applied to two other methods developed for multiple analog fault diagnosis. The dominant differences among these three methods are the distinct fault diagnosis equations resulting from distinct circuit analysis methods and distinct excitation and measurement methods. The method proposed in this paper can be classified as the fault verification method under the category of Simulation-after-Test (SAT) [2], which can provide the exact solution to the circuit parameters and can be applied to detect large parameter changes when the number of independent measurements are greater than the number of faults in the CUT. In Section 2, Kirchhoff current law (KCL) is applied to each circuit node, together with the constitutive equations for all circuit parameters without admittance description, to obtain the modified nodal equation. Circuit topology is comprehensively described by two structural matrices, and Woodbury formula is used to construct the fault diagnosis equation. Fault diagnosis equation relates the limited measured circuit outputs with the faulty parameters in a linear way. In Section 3, a recently developed method for minimum size ambiguity group locating technique based on QR factorization is applied to detect and identify the multiple faults. Detailed procedure and flow-chart of a fault diagnosis program are given. Section 4 provides example circuits to demonstrate the proposed method. The results are compared with those obtained by the method in [9]. The demonstrated methodology is also applied to develop two new methods for multiple fault diagnosis in Section 5. Finally, brief conclusions are drawn in Section 6.

2. FAULT DIAGNOSIS EQUATION

Generally, the circuit topology as well as its parameters' nominal values are known. Consider a continuous-time, time-invariant, strongly connected, linear circuit with $n+1$ nodes and p parameters. The $(n+1)^{th}$ node , denoted by zero, is assigned to be the grounded reference node while the remaining n nodes are ungrounded. All p parameters are divided into two categories: one is parameters which have admittance description such as conductance, capacitor and voltage-controlled-current source, another is parameters which have not admittance description such as impedance, inductor, current-controlled-source, operational amplifier, etc..

Applying the KCL to each circuit node one can obtain n equations with variables being nodal voltages and parameter currents. Constitutive equations in terms of nodal voltages and parameter currents, which define the characteristics of all parameters without admittance description, are appended to the above n KCL-based equations, thus the system's equation are constructed in the following form:

$$T_g X_g = W_g \tag{1}$$

where T_g is a gxg coefficient matrix consisting of circuit parameters, X_g is a $gx1$ solution vector of node voltage and parameter currents, and W_g is a $gx1$ excitation vector composed of independent current and voltage sources, and initial conditions of capacitors and inductors. The first n rows in T_g, X_g and W_g correspond to n nodes. The resulting system equation (1) is called the ***modified nodal equation*** in [10]. Note that $g=n$ for normal nodal analysis of a circuit in which all parameters have admittance description, and $g>n$ for modified nodal analysis of a circuit in which some parameters have non-admittance description. Provided that the circuit functions in a stable state, the parametric values of nodal voltages and parameter currents will be finite and unique. The coefficient matrix T_g is non-singular since the circuit is a strongly connected network.

One important fact about circuit topology is that each parameter, say $h_v \ (v = 1, 2, \ldots, p)$, can be located by at most 4 circuit nodes as indicated in Fig.1: 2 input nodes k_v and l_v, and 2 output nodes i_v and j_v. The current orientations are also indicated in Fig.1. For 2-terminal parameters such as resistor and capacitor, the input nodes will be the same as the output nodes: $k_v = i_v$ and $l_v = j_v$. Based on this fact, the circuit topology can be completely described by two gxp structural matrices P and Q which are defined as follows:

$$\begin{aligned} P &= \lfloor p_1 \ p_2 \ \ldots p_p \rfloor = [e_{i_1} - e_{j_1} \ \ e_{i_2} - e_{j_2} \ \ \ldots \ \ e_{i_p} - e_{j_p}] \\ Q &= \lfloor q_1 \ q_2 \ \ldots \ q_p \rfloor = [e_{k_1} - e_{l_1} \ \ e_{k_2} - e_{l_2} \ \ \ldots \ \ e_{k_p} - e_{l_p}] \end{aligned} \tag{2}$$

where e_v represents a $gx1$ vector of zeros except for the v^{th} entry, which is equal to one, and p_v and q_v represent $gx1$ vectors describing the locations of output nodes and input nodes, respectively. Matrices P and Q are only determined by the locations, not the values of the circuit parameters. The columns of matrix P correspond to the locations of the output nodes of circuit parameters while the columns of matrix Q correspond to the locations of the input nodes of circuit parameters.

Another important fact is that most parameters in linear circuits will enter the coefficient matrix T_g in the symbolic form

$$\begin{array}{c|cc} & k_v & l_v \\ \hline i_v & h_v & -h_v \\ j_v & -h_v & h_v \end{array} \tag{3}$$

with the equivalent algebraic representation being

$$\left(e_{i_v} - e_{j_v}\right) h_v \left(e_{k_v} - e_{l_v}\right)^T = p_v \, h_v \, q_v^T \tag{4}$$

where superscript T denotes transpose of matrix or vector. For any grounded node, the corresponding row or column in the symbolic form will be removed together with the corresponding unit vector e_v in the algebraic form. Resistor, inductor, capacitor, dependent sources, and operational amplifier with its negative inverse gain being a parameter are examples of circuit devices described in this way. In this paper, all faulty parameters are restricted to such type of circuit devices.

Apply (1) to fault-free and faulty circuit, respectively, with the same excitation sources to get

$$T_0 X_0 = W_0 \tag{5}$$

$$TX = (T_0 + \Delta T)(X_0 + \Delta X) = W_0 \tag{6}$$

where

$$T = T_0 + \Delta T \tag{7}$$

$$X = X_0 + \Delta X \tag{8}$$

Suppose that the first f of p parameters are faulty and are changed from their nominal values $h_{10}, h_{20}, \ldots, h_{f0}$ to the new values $h_1 = h_{10} + \delta_1,\, h_2 = h_{20} + \delta_2,\, \ldots,\, h_f = h_{f0} + \delta_f$, where $\delta_1, \delta_2, \ldots, \delta_f$ are the parameter deviations and the deviation vector δ is an *fx1* vector:

$$\delta = [\delta_1 \ \delta_2 \ \ldots \ \delta_f]^T \tag{9}$$

Define F as the faulty parameter set, and assume that each faulty parameter $F_v \ (v = 1, 2, \ldots, f)$ is located on intersection of the corresponding rows i_v and j_v and columns k_v and l_v of the coefficient matrix T. The deviation of the coefficient matrices now has the following form:

$$\Delta T = \sum_{v=1}^{f} p_v \, \delta_v \, q_v^T = P_f \ diag \ (\delta) \, Q_f^{\,T} \tag{10}$$

where $diag(\delta)$ is an *fxf* diagonal matrix and P_f and Q_f are *gxf* matrices which contain 0 and ±1 entries:

$$\begin{aligned} P_f &= \lfloor p_1 \ p_2 \ \ldots p_f \rfloor = [e_{i_1} - e_{j_1} \ \ e_{i_2} - e_{j_2} \ \ \ldots \ \ e_{i_f} - e_{j_f}] \\ Q_f &= \lfloor q_1 \ q_2 \ \ldots \ q_f \rfloor = [e_{k_1} - e_{l_1} \ \ e_{k_2} - e_{l_2} \ \ \ldots \ \ e_{k_f} - e_{l_f}] \end{aligned} \tag{11}$$

Note that P_f and Q_f are sub-matrices of P and Q respectively. They can be constructed from P and Q by selecting all columns in P and Q corresponding to faulty parameters.

The solution vector for fault-free circuit is

$$X_0 = [x_{1,0} \ x_{2,0} \ \ldots \ x_{g,0}]^T \tag{12}$$

where subscript *0* indicates that the denoted parameters are for fault-free circuit. Hence the product of Q_f^T and X_0 can be written as

$$\begin{aligned} Q_f^{\,T} X_0 &= [e_{k_1} - e_{l_1} \ \ e_{k_2} - e_{l_2} \ \ldots \ e_{k_f} - e_{l_f}]^T X_0 \\ &= [x_{k_1,0} - x_{l_1,0} \ \ x_{k_2,0} - x_{l_2,0} \ \ldots \ x_{k_f,0} - x_{l_f,0}]^T \\ &= [x_{k_1 l_1,0} \ \ x_{k_2 l_2,0} \ \ldots \ x_{k_f l_f,0}]^T \end{aligned} \tag{13}$$

and it has the physical interpretation of controlling nominal signal values (e.g. voltages) on faulty parameter input terminals. Applying the Woodbury formula [11] in matrix theory

$$\left(A+PS^{-1}V\right)^{-1}=A^{-1}-A^{-1}P\left(S+VA^{-1}P\right)^{-1}VA^{-1} \tag{14}$$

to (7) and (10) with $A=T_0$, $S^{-1}=diag(\delta)$, $P=P_f$ and $V=Q_f^{\ T}$, the inverse of coefficient matrix T has the following form:

$$\begin{aligned} T^{-1} &= \left(T_0+P_f\ diag(\delta)Q_f^{\ T}\right)^{-1} \\ &= T_0^{-1}-T_0^{-1}P_f\left(diag(\delta^{-1})+Q_f^{\ T}T_0^{-1}P_f\right)^{-1}Q_f^{\ T}T_0^{-1} \end{aligned} \tag{15a}$$

The value of $\delta_\nu\ (\nu=1,2,...,f)$ cannot be zero or infinity to meet with the requirement of inverting restrictions in the Woodbury formula. Since δ_ν being zero means fault-free parameter and only faulty parameters will be identified by following fault diagnosis algorithm, we will have only one restriction: δ_ν cannot be infinite, which corresponds to the case of open admittance or short impedance. But open or short faults can be dealt with by ideal switch introduced in modified nodal analysis [10]. Therefore, the proposed method can handle open and short faults as well.

Let us define

$$\begin{aligned} &\beta=[\beta_1\ \beta_2\ ...\beta_n]^T=T_0^{-1}P_f \\ &\gamma=Q_f^{\ T}T_0^{-1}P_f \end{aligned} \tag{16}$$

then (15a) has following form

$$T^{-1}=T_0^{-1}-\beta\left(diag\,(\delta^{-1})+\gamma\right)^{-1}Q_f^{\ T}T_0^{-1} \tag{15b}$$

Since the coefficient matrices T_0 and T are non-singular, the solution vector for faulty circuit X is then obtained using (6) and considering (15b) and (5):

$$\begin{aligned} X &= T^{-1}W_0 \\ &= T_0^{-1}W_0-\beta\left(diag\,(\delta^{-1})+\gamma\right)^{-1}Q_f^{\ T}T_0^{-1}W_0 \\ &= X_0-\beta\left(diag\left(\delta^{-1}\right)+\gamma\right)^{-1}Q_f^{\ T}X_0 \end{aligned} \tag{17}$$

Thus, the deviation vector ΔX can be obtained by (8) considering (17) and (13):

$$\begin{aligned} \Delta X &= X-X_0 \\ &= -\beta\left(diag\left(\delta^{-1}\right)+\gamma\right)^{-1}Q_f^T X_0 \\ &= \begin{bmatrix} \alpha_{11}\ \alpha_{12}\ ...\alpha_{1f} \\ \alpha_{21}\ \alpha_{22}\ ...\alpha_{2f} \\ ...... \\ \alpha_{g1}\ \alpha_{g2}\ ...\alpha_{gf} \end{bmatrix} \begin{bmatrix} x_{k_1 l_1,\,0} \\ x_{k_2 l_2,\,0} \\ \cdots \\ x_{k_f l_f,\,0} \end{bmatrix} \end{aligned} \tag{18}$$

where

$$\begin{aligned} \alpha &= -\beta\left(diag\left(\delta^{-1}\right)+\gamma\right)^{-1} \\ &= \begin{bmatrix} \alpha_{11}\ \alpha_{12}\ ...\alpha_{1f} \\ \alpha_{21}\ \alpha_{22}\ ...\alpha_{2f} \\ ...... \\ \alpha_{g1}\ \alpha_{g2}\ ...\alpha_{gf} \end{bmatrix} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ ... \\ \alpha_g \end{bmatrix} \end{aligned} \tag{19}$$

Usually voltage measurement is easier to carry out and is less invasive to analog circuit properties than current measurement. Therefore, we only use nodal voltage measurement in this paper. Suppose the i^{th} node is accessible for measurement, then by (18):

$$\Delta X_i = [\alpha_{i1}\ \alpha_{i2}\ \dots\ \alpha_{if}][x_{k_1l_1,0}\ x_{k_2l_2,0}\ \cdots\ x_{k_fl_f,0}]^T \tag{20}$$

According to definition of *gxf* matrix α in (19) and (16), matrix α does not dependent on the location of excitation sources. Thus matrix α is invariant when applying the multiple excitation method, i.e., the same coefficients α_{ij} links deviation of measurements ΔX_i and nominal signal values on faulty parameter $x_{k_jl_j}$ independently on the excitation vector applied. After measuring the corresponding nodal voltages on the i^{th} node with m independent excitation vectors $W_e\ (e = 1, 2, \dots, m)$, we then obtain

$$\begin{aligned}
\Delta X_i^{(1)} &= [\alpha_{i1}\ \alpha_{i2}\ \dots \alpha_{if}][x_{k_1l_1,0}^{(1)}\ x_{k_2l_2,0}^{(1)}\ \cdots\ x_{k_fl_f,0}^{(1)}]^T \\
\Delta X_i^{(2)} &= [\alpha_{i1}\ \alpha_{i2}\ \dots \alpha_{if}][x_{k_1l_1,0}^{(2)}\ x_{k_2l_2,0}^{(2)}\ \cdots\ x_{k_fl_f,0}^{(2)}]^T \\
&\dots\dots \\
\Delta X_i^{(m)} &= [\alpha_{i1}\ \alpha_{i2}\ \dots \alpha_{if}][x_{k_1l_1,0}^{(m)}\ x_{k_2l_2,0}^{(m)}\ \cdots\ x_{k_fl_f,0}^{(m)}]^T
\end{aligned} \tag{21}$$

or in a matrix form

$$\begin{aligned}
\Delta X_i^M = \begin{bmatrix} \Delta X_i^{(1)} \\ \Delta X_i^{(2)} \\ \dots\dots \\ \Delta X_i^{(m)} \end{bmatrix} &= \begin{bmatrix} x_{k_1l_1,0}^{(1)} & x_{k_2l_2,0}^{(1)} & \cdots & x_{k_fl_f,0}^{(1)} \\ x_{k_1l_1,0}^{(2)} & x_{k_2l_2,0}^{(2)} & \cdots & x_{k_fl_f,0}^{(2)} \\ \dots\dots & & & \\ x_{k_1l_1,0}^{(m)} & x_{k_2l_2,0}^{(m)} & \cdots & x_{k_fl_f,0}^{(m)} \end{bmatrix} \begin{bmatrix} \alpha_{i1} \\ \alpha_{i2} \\ \cdots \\ \alpha_{if} \end{bmatrix} \\
&= X_b^{MF} \alpha_i
\end{aligned} \tag{22}$$

where superscript M denotes the set of multiple excitations and m is the number of these excitations. The single measurement node can be one of the nodes used for multiple excitation method, then the total number of accessible nodes should be m. Assume that $f \le m - 1 \le p$, then the coefficient matrix X_b^{MF} has more rows than columns thus to guarantee the uniqueness of solution to (22) with verification. Equation (22) establishes the linear relationship between the measured responses of the faulty circuit ΔX_i^M and the faulty parameter deviations δ (since vector α_i is a linear function of δ according to (19)). Therefore (22) is called **fault diagnosis equation**, the coefficient matrix X_b^{MF} is called **fault diagnosis matrix**.

3. FAULT DIAGNOSIS

Testability is not the focus of this paper. We assume that the given measurement set can give at least one finite solution to circuit parameters.

3.1. *Fault detection*

As the first stage of fault diagnosis, fault detection is easily implemented. If the measurement deviation vector $\Delta X_i^{\;M}$ in the fault diagnosis equation is a zero vector, obviously the CUT is judged as fault-free for the given excitation and measurement sets. Otherwise, at least one fault is judged detected by the given measurement set.

3.2. *Fault identification*

To identify the faulty parameters, first let us analyze the fault diagnosis equation. The left-side of (22) is a known vector from measurements, the right side is the product of an unknown coefficient matrix $X_b^{\;MF}$ and an unknown solution vector α_i. According to (13), matrix $X_b^{\;MF}$ is determined by faulty parameter locations and X_0, solution vector for fault-free circuit. Hence the columns in $X_b^{\;MF}$ represent the differences between the nominal values of nodal voltages or parameter currents across the 2 input nodes of the faulty parameters. Although we do not know matrix $X_b^{\;MF}$, but we really know all of the nodal voltages and parameter currents in fault-free circuit! Similar as in (13), we construct a new *mxp* matrix $X_b^{\;MP}$ as follows

$$\begin{aligned} Q^T X_0 &= [e_{k_1} - e_{l_1}\ \ e_{k_2} - e_{l_2}\ \ \ldots\ e_{k_p} - e_{l_p}]^T X_0 \\ &= [x_{k_1,0} - x_{l_1,0}\ \ x_{k_2,0} - x_{l_2,0}\ \ \ldots\ x_{k_p,0} - x_{l_p,0}]^T \\ &= [x_{k_1 l_1,0}\ \ x_{k_2 l_2,0}\ \ \ldots\ x_{k_p l_p,0}]^T \end{aligned} \tag{23}$$

$$X_b^{\;MP} = \begin{bmatrix} x^{(1)}_{k_1 l_1,0} & x^{(1)}_{k_2 l_2,0} & \cdots & x^{(1)}_{k_p l_p,0} \\ x^{(2)}_{k_1 l_1,0} & x^{(2)}_{k_2 l_2,0} & \cdots & x^{(2)}_{k_p l_p,0} \\ \ldots\ldots & & & \\ x^{(m)}_{k_1 l_1,0} & x^{(m)}_{k_2 l_2,0} & \cdots & x^{(m)}_{k_p l_p,0} \end{bmatrix} \tag{24}$$

where superscript *P* denotes the set of all circuit parameters. Each column of $X_b^{\;MP}$ corresponds to one circuit parameter. Apparently, fault diagnosis matrix $X_b^{\;MF}$ is a sub-matrix of $X_b^{\;MP}$, which can be constructed by collecting all columns in $X_b^{\;MP}$ corresponding to the faulty parameters. Matrix $X_b^{\;MF}$ has more rows than columns whereas $X_b^{\;MP}$ has less rows than columns due to the restriction $f \le m-1 \le p$.

For the purpose of fault identification, we need to find out which set or sets of columns in $X_b^{\;MP}$ can satisfy the fault diagnosis equation, i.e., the dependency between $\Delta X_i^{\;M}$ and the desired set(s) of columns in $X_b^{\;MP}$. It is very possible that there are more than one qualifying sets, so we regulate that the minimum size of column set satisfying fault diagnosis equation will be the desired coefficient matrix in fault diagnosis matrix. One obvious way is to have a combinatorial search through all columns in $X_b^{\;MP}$, which is the traditional way in fault verification method [2] and requires the number of operation $O\left(\sum_1^f \binom{p}{i}\right)$ for limited faults among *p* parameters, thus it is computationally costly. More efficient method for fault identification is expected to reduce the computational cost. Our idea is to transform fault identification problem into a mathematical problem: locating the minimum size ambiguity group which satisfy the fault diagnosis equation. Ambiguity group is defined as a set of parameters corresponding to linearly dependent columns of X_b^{MP} which in general case does not give a unique solution in fault identification. However, in this work we will show how the set of faulty parameters can be identified by finding ambiguity groups.

In [12], a method was developed to locate the minimum size ambiguity groups by using a linear combination matrix *C* (which will be introduced later) with minimum number of non-zero entries. In this paper we modify the method in [12] to identify dependence of the measurement vector ΔX_i^M on a subset of columns from X_b^{MP}. Gaussian Elimination step is introduced, and minimum size ambiguity group is located by identifying the column with minimum number of non-zero entries in the linear combination matrix *C*. The three steps, Gaussian elimination, QR factorization and swapping performance are detailed next.

3.2.1. Gaussian Elimination

First let us denote an augmented *mx(p+1)* matrix B_S as the concatenation of the vector ΔX_i^M and the matrix X_b^{MP}:

$$B_S = [\Delta X_i^M \;\; X_b^{MP}] \tag{25}$$

Then we will normalize the first column of matrix B_S to have a unit in its first row,

$$\hat{B}_S(i,1) = \frac{B_S(i,1)}{B_S(1,1)}, \quad i = 1,2,...,m. \tag{26}$$

If the first entry of matrix B_s, *$B_s(1,1)$* happens to be zero, just permutes the rows of B_s so that the first entry *$B_s(1, 1)$* is non-zero. Such a nonzero entry must exist since ΔX_i^M is a non-zero vector for faulty circuit. Eliminate the remaining entries in the first row of matrix B_S by performing a similar operations to Gaussian elimination as follows:

$$\hat{B}_S(i,j) = B_S(i,j) - \frac{B_S(i,1)}{B_S(1,1)} B_S(1,j), \quad i = 1,2,...,m; j = 2,3,...,p+1 \tag{27}$$

Finally we obtain *mx(p+1)* matrix $\hat{B}_S$ in the following form:

$$\hat{B}_S = \begin{bmatrix} 1^{1\times 1} & 0^{1\times p} \\ (\Delta\hat{X}_i)^{(m-1)\times 1} & B^{(m-1)\times p} \end{bmatrix} \tag{28}$$

where the superscript represents the size of a vector or a matrix. Matrix *B* is obtained from X_b^{MP} after elimination of dependence on ΔX_i^M and is called **verification matrix**. The dependency of the desired columns of matrix *B* surely indicate the dependency between ΔX_i^M and the desired columns of matrix X_b^{MP}. Thus we can only concentrate on the dependency among the columns of the verification matrix *B*.

3.2.2. QR factorization

The rank of B determines a maximum number of faults that can be uniquely identified by solving the fault diagnosis equation. Because $m-1<p$, B can be decomposed into two linearly dependent sub-matrices as follows

$$B = [B_1 \ B_2] = B_1 [I \ C] \tag{29}$$

$$B_2 = B_1 C \tag{30}$$

Where $(m-1)xr$ matrix B_1 has the full column rank equal to the rank r of the matrix B, and $r \times (p-r)$ matrix C is called **linear combination matrix** whose columns expand a set of basis columns from B_1 into the corresponding columns of B_2. Note that the selection of independent columns of B_1 is not unique and is an important issue in solving the fault diagnosis equation in the presence of ambiguities. Different partitions define different linear combination matrices C.

Since an ambiguity group is a set of circuit parameters corresponding to linearly dependent columns of B, we define a canonical ambiguity group as a minimal set of parameters corresponding to linearly dependent columns of B. This means that if any single parameter is removed from the canonical ambiguity group, then the remaining set corresponds to independent columns of B and can be uniquely solvable. A combination of canonical ambiguity groups with at least one common element was defined as ambiguity cluster.

To efficiently deal with fault verification problem, we will look for a partition (29) with the matrix C in a **minimum form**, which is defined as such a matrix that one or several of its columns have the maximum number of entries equal to zero. Thus, we can get the minimum number of columns in X_b^{MP} satisfying the fault diagnosis equation (22). The corresponding partition (29) is called a canonical form of the fault diagnosis equation. Notice that according to fault verification principles [2] it is enough to find a single entry in one column of C equal to zero to solve the fault diagnosis equation. This column and all rows with non-zero entries will correspond to the faulty parameters as indicated by the element of co-basis B_2 and elements of basis B_1, respectively.

In this paper we will refer to a numerically robust algorithm based on the *QR* factorization [12], which can find a numerically stable solution of over determined system of linear equations that minimizes the least square error. Fault diagnosis equation use more measurements than the number of unknown variables in order to be able to find a unique solution as well as to compensate for the measurement errors and noise of the measurement equipment. At least one extra measurement is needed to verify the fault selection hypothesis.. As a result of the *QR* factorization of $(m-1)xp$ verification matrix B, we obtain:

$$BE = QR \tag{31}$$

where E is pxp column selection matrix, Q is $(m-1)x(m-1)$ orthogonal matrix, and R is $(m-1)xp$ upper triangular matrix. Each column of matrix E has only one nonzero entry, which is equal to one. Matrix product BE represents a permutation of the original columns of the verification matrix B. Matrix R has its rank equal to the rank of matrix B. Since R is an upper triangular matrix and $m-1<p$, R can be written as

$$R = \begin{bmatrix} R_1 & R_2 \\ 0 & 0 \end{bmatrix} \tag{32}$$

where R_1 is $r \times r$ upper triangular and has its rank equal to the rank of the verification matrix B.

The following theorem in [12] provides a basis for a numerically efficient approach to finding the ambiguity groups.

Theorem:

A linear combination matrix C can be numerically obtained from the QR factorization of the verification matrix B using

$$C = R_1^{-1} R_2 \tag{33}$$

3.2.3. Swapping performance

A single QR run cannot guarantee that the matrix *C* will be obtained with one or several of its columns having the maximum number of zero entries if the proper basis is not selected. To find the minimum form partition, we have to swap one parameter of the basis with one parameter of the co-basis in the ambiguity cluster in order to increase number of nonzero entries in *C*. Note that swapping parameters of the basis and the co-basis can be performed independently in different ambiguity clusters, since different clusters have mutually disjoint sets of parameters.

Lemma 1 [12]:

The necessary condition for swapping to increase the number of zero entries in C is that the columns of basis and co-basis to be swapped have a singular 2x2 sub-matrix of nonzero entries.

Let us consider a linear combination matrix *C* with a *2x2* singular sub-matrix $\begin{bmatrix} c_{jk} & c_{jm} \\ c_{ik} & c_{im} \end{bmatrix}$ with all nonzero entries. If we swap the *j*th element of the basis with *k*th element of the co-basis, then after swapping, the *k*th column of *C* changes to

$$C_k = -\frac{1}{c_{jk}}\left[c_{1k} \;\; c_{2k} \;\cdots\; 1 \cdots \; c_{rk}\right]^T \tag{34}$$

In addition, all other columns of matrix *C* will be equal to

$$C_n = \left[c_{1n} - \frac{c_{jn}c_{1k}}{c_{ik}} \;\; c_{2n} - \frac{c_{jn}c_{2k}}{c_{ik}} \cdots \frac{c_{jn}}{c_{ik}} \cdots \; c_{rn} - \frac{c_{jn}c_{rk}}{c_{ik}} \right]^T \tag{35}$$

Such that all zero locations in the *k*th column of *C* will remain zero as they were in the original *C*. However, as can be deducted from (34), a nonzero location c_{im} in row *i* and column *m* will become zero. Any column of *C* with zero entries form an ambiguity group *F* and has to be consider for further processing. Since ambiguities may exist in the original matrix X_b^{MP} then *F* contains all faults in the CUT only if the corresponding columns in X_b^{MP} are independent. Hence we can formulate the following lemma:

Lemma 2:

A necessary condition for an ambiguity group F of the linear combination matrix C to contain the set of all faults in the tested circuit is that the rank of the corresponding columns in matrix X_b^{MP} *is equal to the cardinality of F.*

$$rank(X_b^{MP}) = card(F) \tag{36}$$

Thus according to Lemma 2 any ambiguity group of the verification matrix which do satisfy (36) needs to be further analyzed.

The number of operations required for Gaussian elimination step is $O(p^2)$, $O(p^3)$ for QR factorization and $O((p-r)^3)$ for swapping performance, so the computational cost of the proposed method is $O(p^3)$.

3.3. Parameter evaluation

After location of the faulty parameters, the invariant vector α_i can be uniquely solved from (22):

$$\alpha_i = \left(\left(X_b^{MF}\right)^T X_b^{MF}\right)^{-1} \left(X_b^{MF}\right)^T \Delta X_i^M \tag{37}$$

Then, the deviation vector δ can be exactly computed by

$$\delta = \alpha_i \ rdivide\left(-\beta_i - \alpha_i \gamma\right) \tag{38}$$

where *rdivide* is an element-by-element division of two vectors. Additionally, The other parameters in the faulty circuit can be obtained from the construction process of fault diagnosis equation. For example, the deviation vector ΔX can be obtained by (18) considering (16), then the solution vector for faulty circuit *X* can be obtained from (8). Alternatively, vector *X* can be solved from (6) by inverting its coefficient matrix *T*, obtained by (7) and (10). In one word, everything about the faulty circuit can be known.

3.4. Algorithm for fault diagnosis

A flow diagram of a computer program which implements the fault diagnosis discussed above is shown in Fig. 2. Since most of the phases of the algorithm are self-evident from the flow diagram, only some phases are detailed in this section.

In Phase 1, since nominal values of circuit parameters are known and all nodal voltages in fault-free circuit can be solved by (5), we only need to measure the nodal voltages of the i^{th} node in the CUT under multiple excitation method to obtain measurement deviation vector ΔX_i^M.

In Phase 5, *F* denotes one suspicious fault set and ***min(size(F))*** represents a scalar which is equal to the minimum size of all suspicious fault sets.

In Phase 6, if several suspicious fault sets have the same minimum size, ***min(size(F))***, select one of them arbitrarily for analysis. Only one parameter in the selected *F* is from the co-basis and the remaining parameters from the basis. Swap that co-basis parameter which corresponds to column *k* in matrix *C* with one of basis parameters which corresponds to row *j* in the matrix *C*. By (34) and (35), all zero entries in the column *k* of matrix *C* will be hold after swapping while new zero-entry will appear in another column of new matrix *C*, thus the new value of ***min(size(F))*** will be equal to, or less than the old value before swapping.

There are two rules for swapping. One is that according to Lemma 1 row *j* is selected with nonzero c_{jk} on the intersection of row *j* and column *k* of matrix *C*. Another rule is that if one parameter in the current basis has been swapped into the basis by the previous swapping operation, then this element will not be considered during the later swapping operation. Usually *m-1* is far less than *p*, and the rank of *rx(p-r)* matrix *C*, *r* is not greater than *m-1*, thus there are far less basis parameters than co-basis parameters. The comprehensive swapping between the co-basis parameter *k* and the basis parameters are very limited, as a result of the two swapping principles.

Phase 12 through 15B is used for verification. One or several suspicious fault sets with minimum size are used to compute the deviation vector ΔX. If a computed vector matches the real measured vector ΔX_i^M, the corresponding fault set *F* is our final solution to faulty parameters. Otherwise, we discard this set, and turn to the adjoint suspicious fault sets recorded in Phase 9. Verification in this phase continues until find out at least one qualified solution to faulty parameters. Otherwise, the CUT is concluded as un-solvable because the restriction $f \le m-1$ is not satisfied.

4. EXAMPLE CIRCUITS

Example 1. The example circuit 4 in [9] is used here in order to demonstrate the improvement in efficacy by the method proposed in this paper. There are 6+1 nodes, 11 conductances, 2 voltage-controlled-current sources in the CUT, where G_1=1S, G_2=1S, G_3=2S, G_4=1S, G_5=0.5S, G_6=2S, G_7=1S, G_8=0.5S, G_9=2S, G_2=1S, G_{11}=0.5S, i_s=1A. Suppose that G_3 and G_9 have deviations $\Delta G_3 = -1\ S$ and $\Delta G_9 = 2\ S$, respectively. Node *{1}* is the single measurement node. The single current source i_s is applied between ground and three accessible nodes *{1, 3, 6}* respectively under multiple excitation method. Thus, *n=6*, *p=13*, *m=3*, *f=2* and $f \le m-1 \le p$. The measurement deviation in Phase 1 of algorithm is

$$\Delta X_i^M = \begin{bmatrix} 0.2248 \\ -2.1536 \\ -1.2544 \end{bmatrix}$$

In Phase 4, verification matrix *B* is obtained after Gaussian elimination as

$$B = \begin{bmatrix} 5.3827 & -4.1975 & 1.1852 & -1.8272 & -0.6420 & 0.7531 & 0.1111 & -0.9877 & 0.7407 & -0.3580 & -0.2469 & 0.6420 & 1.0988 \\ 3.3827 & -2.1975 & 1.1852 & 0.1728 & 1.3580 & -0.2469 & 1.1111 & -0.9877 & 0.7407 & -1.3580 & -0.2469 & -1.3580 & 2.0988 \end{bmatrix}$$

and the linear combination matrix *C* is obtained as

$$C = \begin{bmatrix} 0.2500 & -0.2500 & -0.7500 & 0.0911 & 0.0911 & -0.2083 & 0.1563 & -0.1432 & -0.0521 & 0 & 0.2995 \\ 0.2500 & 0.7500 & 0.2500 & -0.4089 & 0.5911 & -0.2083 & 0.1562 & -0.6432 & -0.0521 & -1.0000 & 0.7995 \end{bmatrix}$$

with permutation vector *E={1, 5, 3, 4, 2, 6, 7, 8, 9, 10, 11, 12, 13}*. Thus the basis parameters are *{1, 5}* and co-basis parameters *{3, 4, 2, 6, 7, 8, 9, 10, 11, 12, 13}*. The only suspicious fault set *{5, 12}* is from the 10th column of *C*, but it does not satisfy Lemma 2.

Swapping the first basis parameter *{1}* with the first co-basis parameter *{3}*, we will obtain the new matrix *C* as

$$C = \begin{bmatrix} 4.0000 & -1.0000 & -3.0000 & 0.3646 & 0.3646 & -0.8333 & 0.6250 & -0.5729 & -0.2083 & 0 & 1.1979 \\ -1.0000 & 1.0000 & 1.0000 & -0.5000 & 0.5000 & 0.0000 & -0.0000 & -0.5000 & 0.0000 & -1.0000 & 0.5000 \end{bmatrix}$$

Totally there are three suspicious fault sets *{3, 8}*, *{3, 9}* and *{3, 11}*, and ***min(size(F))***=2. Since we cannot reduce ***min(size(F))*** any more by swapping, we conclude that these three fault sets are our candidates for verification in Phase 13 through 15B.

For fault set *{3, 8}*, the fault diagnosis equation is

$$\begin{bmatrix} 0.2248 \\ -2.1536 \\ -1.2544 \end{bmatrix} = \begin{bmatrix} -0.1304 & 0.6957 \\ 2.4348 & -7.6522 \\ 1.9130 & -4.8696 \end{bmatrix} \alpha_i$$

with its unique solution vector by (37) $\alpha_i = [0.3191 \quad 0.3830]^T$. By (38), the deviations of G_3 and G_8 are

$$\begin{bmatrix} \Delta G3 \\ \Delta G8 \end{bmatrix} = \begin{bmatrix} -1.0000 \\ 0.2647 \end{bmatrix}$$

The computed nodal voltage deviations on node *{1}* is

$$\Delta X_i^{computed} = \begin{bmatrix} 0.2248 \\ -2.1536 \\ -1.2544 \end{bmatrix}$$

which is equal to the measured vector ΔX_i^M. Thus, we conclude that fault parameters are G_3 and G_8 with $\Delta G_3 = -1 \ S$ and $\Delta G_8 = 0.2647 \ S$ respectively.

For fault set *{3, 9}*, the fault diagnosis equation is

$$\begin{bmatrix} 0.2248 \\ -2.1536 \\ -1.2544 \end{bmatrix} = \begin{bmatrix} -0.1304 & -0.5217 \\ 2.4348 & 5.7391 \\ 1.9130 & 3.6522 \end{bmatrix} \alpha_i$$

with the deviations of G_3 and G_9 are

$$\begin{bmatrix} \Delta G3 \\ \Delta G9 \end{bmatrix} = \begin{bmatrix} -1.0000 \\ 2.0000 \end{bmatrix}$$

The computed vector of nodal voltage deviations on node *{1}* is also equal to the measured vector ΔX_i^M. We conclude that fault parameters are G_3 and G_9 with $\Delta G_3 = -1 \ S$ and $\Delta G_9 = 2.0000 \ S$ respectively.

For fault set *{3, 11}*, similar conclusion is made that fault parameters are G_3 and G_{11} with $\Delta G_3 = -1 \ S$ and $\Delta G_{11} = 1.6364 \ S$ respectively.

Totally we have three solutions to the faulty parameters for the given measurements. To exactly identify the faulty parameters in the CUT, more measurements are needed, which will be demonstrated in next example.

The accessible nodes are reduced to 3 in the proposed method comparing with at least 4 accessible nodes in [9]: node *{1, 6}* for multiple excitation method and node *{3, 4}* for measurement of branch voltage of G_6. The selection and assumption of one fault-free parameter with corresponding measurement of its branch voltage used in decomposition method in [9] is removed, which is a notable improvement.

Example 2. An active low-pass filter [15] shown in Fig. 4a is provided to illustrate the approach proposed in the paper. The example circuit has 20 nodes and 22 resistors, 4 capacitors, and 8 amplifiers with the following nominal values (all resistors in $k\Omega$ and capacitors in μF): R1=0.182, C2=0.01, R3=1.57, R5=2.64, R6=10.0, R7=10.0, R9=100.0, R10=11.1, R11=2.64, C12=0.01, R14=5.41, R15=1.0, R17=1.0, C18=0.01, R19=4.84, R21=2.32, R22=10.0, R23=10.0, R25=500.0, R26=111.1, R27=1.14, R28=2.32, C29=0.01, R31=72.4, R32=10.0, R34=10.0. The current source is $j(t)=1.0\cos(2000t)A$. All the operational amplifiers are modeled by the circuit in Fig. 4b.

Assume that the faulty parameters are R6 which was changed from $10.0k\Omega$ to $20.0k\Omega$ and R26 changed from $111.1k\Omega$ to $75.0k\Omega$. The corresponding admittance deviations are $\Delta G6 = 1/20000 - 1/10000 = -5.0e-5/\Omega$ and $\Delta G26 = 1/75000 - 1/111100 = 4.3324e-6/\Omega$. The single measurement node is node {2}, and single current source is applied between ground and nodes {1, 2, 7, 17, 19}. Thus *n=19*, *p=42*, *f=2*, *m=5* and restriction $f \le m-1 \le p$ is satisfied. The measured deviation vector is

$$\Delta X_i^M = \begin{bmatrix} -3.4938\text{e}-003+1.3508\text{e}-002\text{i} \\ -3.5511\text{e}-003+1.3729\text{e}-002\text{i} \\ 2.6940\text{e}-001+7.0256\text{e}-002\text{i} \\ -5.1196\text{e}-014+2.1975\text{e}-013\text{i} \\ -3.5511\text{e}-003+1.3729\text{e}-002\text{i} \end{bmatrix}$$

In Phase 4, a *4x38* linear combination matrix *C* is obtained after Gaussian elimination and QR factorization with the basis parameters *{3, 30, 7, 17, 5}* and co-basis parameters *{6, 1, 8, 9, 10, 11, 12, 13, 14, 15, 16, 4, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 2, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42}*. By lemma 2, two suspicious fault sets are identified *{5, 17}* and *{4, 17}* with ***min(size(F))****=2*.

Since no swapping can reduce ***min(size(F))*** any more, we obtain two suspicious fault sets *{5, 17}*, and *{4, 17}*.

Fault set *{4, 17}*, correspond to parameters *{R6, R26}* in the CUT. The fault diagnosis equation is

$$\begin{bmatrix} -3.4938\text{e}-003+1.3508\text{e}-002\text{i} \\ -3.5511\text{e}-003+1.3729\text{e}-002\text{i} \\ 2.6940\text{e}-001+7.0256\text{e}-002\text{i} \\ -5.1196\text{e}-014+2.1975\text{e}-013\text{i} \\ -3.5511\text{e}-003+1.3729\text{e}-002\text{i} \end{bmatrix} = \begin{bmatrix} -1.1044\text{e}+001+1.3172\text{e}+002\text{i} & -5.5322\text{e}+002-4.9887\text{e}+001\text{i} \\ -1.1225\text{e}+001+1.3388\text{e}+002\text{i} & -5.4184\text{e}+002-5.0614\text{e}+001\text{i} \\ 2.6279\text{e}+003+2.2562\text{e}+002\text{i} & -9.4639\text{e}+002-5.6622\text{e}+001\text{i} \\ -9.0468\text{e}-011+1.0790\text{e}-009\text{i} & 1.0101\text{e}+000+6.4128\text{e}-011\text{i} \\ -1.1225\text{e}+001+1.3388\text{e}+002\text{i} & -5.4183\text{e}+002-5.0614\text{e}+001\text{i} \end{bmatrix} \alpha_i$$

with $\alpha_i = [1.0404\text{e}-004-1.7802\text{e}-005\text{i} \quad -2.2349\text{e}-014-1.0800\text{e}-013\text{i}]^T$. By (38), the deviations of G_6 and G_{26} are

$$\begin{bmatrix} \Delta G6 \\ \Delta G26 \end{bmatrix} = \begin{bmatrix} -5.0000\text{e}-005+6.3277\text{e}-021\text{i} \\ 4.3324\text{e}-006+3.6403\text{e}-012\text{i} \end{bmatrix} \approx \begin{bmatrix} -5.0000e-005 \\ 4.3324e-006 \end{bmatrix}$$

The computed vector of nodal voltage deviations on node *{2}* is also equal to the measured vector ΔX_i^M. We conclude that fault parameters are G_6 and G_{26} with $\Delta G_6 = -5e-5\ S$ and $\Delta G_{26} = 4.3324\ e-6\ S$.

Fault set *{5, 17}* correspond to parameters *{R7, R26}* in the CUT. By (38), the deviations of G_7 and G_{26} are

$$\begin{bmatrix} \Delta G7 \\ \Delta G26 \end{bmatrix} = \begin{bmatrix} 9.9075\text{e}-005-1.5686\text{e}-007\text{i} \\ -7.7897\text{e}-013+3.9789\text{e}-012\text{i} \end{bmatrix}$$

Obviously, R7 should not have imaginary part even in the faulty condition. Thus, we discard this fault set.

In conclusion, we identify only one faulty parameter set *{R6, R26}* with their deviations in the CUT, which is the exact faulty condition in the CUT. With increased number of measurements, the suspicious faulty parameter sets are reduced to a unique solution set which matches the real condition.

5. GENERALIZED APPLICATIONS

The mechanism demonstrated in the proposed method can be generalized as follows. First construct the fault diagnosis equation based on circuit analysis to relate the limited measured circuit responses with the faulty parameters in a linear way, then apply the ambiguity group locating technique to identify the faulty parameters through three steps: Gaussian elimination, QR factorization and swapping performance. Finally evaluate all parameter values of the faulty circuit based on the analysis of the fault diagnosis equation. Two new methods sharing the same mechanism were proposed recently for multiple fault diagnosis in linear analog circuits [13-14].

5.1. Method 1

This method is described in detail in [13]. Starting from (5) and (6), we can obtain

$$T_0\,\Delta X = -\Delta T\,X \tag{39}$$

Then ΔX is computed by

$$\Delta X = -T_0^{-1}\,\Delta T\,X \tag{40}$$

Let us denote

$$\Delta W = -\Delta T\,X \tag{41}$$

where *gx1* vector ΔW represents the changes in excitations caused by faulty parameters and we call it the **faulty excitations**. The corresponding nodes or parameters are faulty. Similarly, nodes or parameter with zero faulty excitations are fault-free. The equation (40) is simplified as

$$\Delta X = T_0^{-1}\,\Delta W \tag{42}$$

Since only a few parameters are faulty, in which case ΔW has the form

$$\Delta W = \begin{bmatrix} 0 \\ \Delta W^F \\ 0 \end{bmatrix} \tag{43}$$

Assuming that the first *m* elements of *X* can be measured we obtain

$$\begin{bmatrix} \Delta X^M \\ \Delta X^{G-M} \end{bmatrix} = T_0^{-1} \begin{bmatrix} 0 \\ \Delta W^F \\ 0 \end{bmatrix} \tag{44}$$

where *G* indicates the set of all equations, *M* the set of measurements. Hence,

$$\Delta X^M = B_{MF}\,\Delta W^F \tag{45}$$

where

$$T^{-1} = \begin{bmatrix} B_{M1} & B_{MF} & B_{M2} \\ B_{N-M,1} & B_{N-M,F} & B_{N-M,2} \end{bmatrix} \tag{46}$$

$$B_M = \begin{bmatrix} B_{M1} & B_{MF} & B_{M2} \end{bmatrix} \tag{47}$$

Fault diagnosis equation (45) has to be satisfied when the set F includes all circuit excitations associated with faulty parameters in the faulty circuit. The columns in B_{MF} correspond to faulty nodes or faulty parameters in the circuit. Our aim is to find out the sets of columns in matrix B_M that satisfy equation (45) with the minimum number of faults, that is, vector ΔW^F has the minimum number of nonzero values.

The same ambiguity group locating technique discussed in Section 3.2 can be applied to identify the minimum form ambiguity group after constructing a *mx(g+1)* matrix B_s as follows.

$$B_S = [\Delta X^M \;\; B_M] \tag{48}$$

After location of faulty excitations, the deviation of the faulty excitation vector can be derived by solving (45),

$$\Delta W^F = \left(B_{MF}{}^T B_{MF}\right)^{-1} B_{MF}{}^T \Delta X^M \tag{49}$$

Then the deviation of the excitation vector can be obtained by filling out the remaining elements with zeros to get ΔW in (43). The deviation of the solution vector ΔX can be obtained by (42), solution vector for faulty circuit X can be obtained by (8). Combining (10) into (41),

$$\Delta W = -\Delta T\, X = -P_f\, diag(\delta) Q_f^T\, X = X_{inc}\,\delta \tag{50}$$

where

$$X_{inc} = -P_f\, diag(Q_f^T X) \tag{51}$$

Assuming that k of p parameters are faulty and f of g excitations are faulty, k is no greater than f because some parameters maybe locate between two ungrounded nodes. We re-arrange the equation (50) as follows:

$$X_{inc}{}^{f,k}\;\delta^k + X_{inc}{}^{f,p-k}\; 0^{p-k} = (\Delta W)^f \tag{52a}$$

$$X_{inc}{}^{n-f,k}\;\delta^k + X_{inc}{}^{n-f,p-k}\; 0^{p-k} = 0^{n-f} \tag{52b}$$

Here the superscript indicates the size of the matrix or vector. The equation (52b) is worth consideration. Obviously with nonzero values of δ^k, $X_{inc}{}^{n-f,k}$ must be $0^{n-f,k}$ with probability equal to 1. We can obtain the position of faulty elements δ^k from the solution of equation (52b) as follows:

Lemma 3:

The k faulty parameters are included in the parameter set whose corresponding columns have all zero entries in the matrix $X_{inc}{}^{n-f,p}$.

The deviations of faulty parameters then can be derived by solving (52a)

$$\delta = \left(\left(X_{inc}{}^{f,k}\right)^T X_{inc}{}^{f.k}\right)^{-1} (X_{inc}{}^{f,k})^T\; (\Delta W)^f \tag{53}$$

5.2. Method 2

This method is discussed in details in [14]. Similar as in the method proposed in this paper, but without Woodbury formula, combining (10) into (6), we get

$$\left(T_0 + P_f\, diag(\delta) Q_f^T\right)\left(X_0 + \Delta X\right) = W_0 \tag{54}$$

After substituting (5) into (54), the following equation is established,

$$\Delta X = -T_0^{-1} P_f \; diag(\delta) Q_f^T X \tag{55}$$

Let us denote a *gxg* matrix S_0 as follows

$$S_0 = [s_1 \; s_2 \; \ldots \; s_g] = -T_0^{-1} \tag{56}$$

where X and $s_v \; (v = 1,2,\ldots,g)$ are *gx1* vectors. Thus the products of S_0 and P_f, Q_f^T and X can be written as

$$\begin{aligned} S_{GF} &= S_0 P_f = S_0 [e_{i_1} - e_{j_1} \;\; e_{i_2} - e_{j_2} \;\; \ldots \; e_{i_f} - e_{j_f}] \\ &= [s_{i_1} - s_{j_1} \;\; s_{i_2} - s_{j_2} \;\; \ldots \; s_{i_f} - s_{j_f}] \\ Q_f^T X &= [e_{k_1} - e_{l_1} \;\; e_{k_2} - e_{l_2} \;\; \ldots \; e_{k_f} - e_{l_f}]^T X \\ &= [x_{k_1} - x_{l_1} \;\; x_{k_2} - x_{l_2} \;\; \ldots \; x_{k_f} - x_{l_f}]^T \end{aligned} \tag{57}$$

where G indicates the set of all modified nodal equations and the **fault set** F represents the set of all the faulty parameters.

Denote an *fx1* vector

$$\lambda_F = diag(\delta) \; Q_f^T X \tag{58}$$

and consider (9) and (57) to get

$$\begin{aligned} \lambda_F &= diag\;(\delta)\; Q_f^T X \\ &= diag\;(\delta)\,[x_{k_1} - x_{l_1} \;\; x_{k_2} - x_{l_2} \;\; \ldots \; x_{k_f} - x_{l_f}]^T \\ &= [\delta_1 (x_{k_1} - x_{l_1}) \;\; \delta_2 (x_{k_2} - x_{l_2}) \;\; \ldots \;\; \delta_f (x_{k_f} - x_{l_f})]^T \end{aligned} \tag{59}$$

Thus (55) can be re-written as

$$\Delta X = S_{GF} \lambda_F \tag{60}$$

Assume that the first m elements of ΔX can be measured and $f \le m - 1 \le p$, we obtain

$$\begin{bmatrix} \Delta X^M \\ \Delta X^{G-M} \end{bmatrix} = \begin{bmatrix} S_{MF} \\ S_{G-M,F} \end{bmatrix} \lambda_F \tag{61}$$

where M represents the set of measurements. Hence, following **fault diagnosis equation** is obtained:

$$\Delta X^M = S_{MF} \, \lambda_F \tag{62}$$

Here S_{MF} is an *mxf* matrix whose columns correspond to the faulty parameters in the circuit. Similarly S_{MP} is an *mxp* matrix whose columns correspond to all of the parameters in the circuit, which is constructed by selecting all the rows corresponding to measurements selected from the following matrix S_{GP},

$$\begin{aligned} S_{GP} &= S_0 P = S_0 [e_{i_1} - e_{j_1} \;\; e_{i_2} - e_{j_2} \;\; \ldots \; e_{i_p} - e_{j_p}] \\ &= [s_{i_1} - s_{j_1} \;\; s_{i_2} - s_{j_2} \;\; \ldots \; s_{i_p} - s_{j_p}] \end{aligned} \tag{63}$$

Construct a *mx(p+1)* matrix B_s as follows,

$$B_S = [\Delta X^M \;\; S_{MP}] \tag{64}$$

Then apply the ambiguity group locating technique from Section 3.2 to identify the minimum form ambiguity group. After location of ambiguity groups in the fault diagnosis equation, we know clearly which parameters in the CUT are faulty. Vector λ_F is then obtained by solving (62):

$$\lambda_F = \left(S_{MF}{}^T S_{MF}\right)^{-1} S_{MF}{}^T \Delta X^M \tag{65}$$

The full vector ΔX can be computed by (60) since matrix S_{GF} and vector λ_F are known now. The solution vector X is consequently determined by (8). Finally the parameter deviations δ can be obtained by solving (59):

$$\delta = \begin{bmatrix} \dfrac{\lambda_1}{x_{k_1} - x_{l_1}} & \dfrac{\lambda_2}{x_{k_2} - x_{l_2}} & \cdots & \dfrac{\lambda_f}{x_{k_f} - x_{k_f}} \end{bmatrix}^T \tag{66}$$

5.3. Comparisons of the three fault verification methods

The dominant feature of these three methods is that all them share the same mechanism discussed at the beginning of Section 5. The differences among the methods are in fault parameter location, mathematical tools, excitations, and measurements and are given in Table 1. Due to different methods of circuit analysis and mathematical tools utilized, distinct fault diagnosis equations are constructed. As a consequence, distinct parameter evaluations are proposed for each method. All of these methods belong to the same category of multiple fault verification in dynamic analog circuits and all of them benefit from efficient ambiguity groups location technique presented in [12].

6. CONCLUSIONS

In this paper, a generalized fault verification approach for dynamic analog circuits was discussed. Fault verification methods intend is to obtain the information about the faulty circuit based on the limited measured responses of the faulty circuit. There are two easily implemented prerequisites: one is that the circuit topology and nominal values of circuit parameters should be known, another is that the number of measurements minus one is not less than the number of faulty parameters. A new method proposed in this paper is used to detect, and locate the multiple faults in a linear analog circuit in frequency domain, then to exactly evaluate the faulty parameter deviations. Applying the Woodbury formula in the matrix theory to the modified nodal analysis, fault diagnosis equation is constructed to establish the relationship between the measured responses and the faulty parameter deviations in a linear way. A numerically robust approach developed recently has been modified to fit the condition stated in this paper in order to implement fault location, i.e., location of the minimum size ambiguity group in the fault diagnosis equation based on QR factorization. Parameter evaluation is then performed from results of the analysis of fault diagnosis equation.

One node for voltage measurement is sufficient for the proposed method although multiple excitations are required for fault location. Although the faulty parameter deviation cannot be infinity, open or short condition can be dealt with well by switches in modified nodal analysis. Therefore, the faults can be parametric or catastrophic. The proposed method is extremely effective for large parameter deviations and a very limited number of accessible nodes used for excitations and measurements. The computation cost for the fault location is on the order of $O(p^3)$, and compares favorably with the combinatorial search traditionally used in fault verification methods which requires the number of operations $O\left(\sum_1^f \binom{p}{i}\right)$. A single fault diagnosis method recently reported in [9] can be seen as a special case of the proposed method.

Example circuits are used to illustrate the proposed method and improvement in the efficacy as compared with [9] is evident. Finally, two new methods for multiple fault diagnosis based on the same methodology are discussed and comparisons among these three methods are given.

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