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GPS signal acquisition using the repeatability of successive code phase measurements

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Abstract This paper introduces a new method for GPS signal acquisition, which is based on the repeatability of successive code phase measurements and the M-of-N search algorithm. The performance of the proposed method in terms of probability of signal detection is similar to that of traditional methods, except that the calculation of the probability of detection does not rely on the noise distribution or the Carrier-to-Noise ratio (C/N_0). The code phase repeatability-based method is presented along with equations for probability of detection and probability of false detection. If the distribution of the noise is known, it also provides an estimate of the C/N_0. The proposed method is illustrated for coherent and non-coherent acquisition and C/N_0 estimation.

Keywords GPS signal acquisition · M-of-N search · Code phase repeatability

Introduction

Many GPS applications require the ability to acquire and track weak signals. An effective approach to acquire weak signals is integration over time, which can be performed coherently or non-coherently. Non-coherent demodulation is generally used for acquisition, since it does not require knowledge of carrier phase and precise carrier frequency, both of which are not available before the signal has been acquired (Ward 1996). It is noted, however, that applications exist that use coherent demodulation for signal acquisition (Soloviev et al. 2004). Coherent integration provides for a narrow pre-detection signal bandwidth, which enhances the acquisition of weak signals in the presence of strong in-band interference. Compared to non-coherent acquisition, coherent acquisition requires a more extensive frequency search, which may not be desirable for fast GPS signal acquisition. Based on the target application, tolerable wait time and available computational resources, GPS receiver designers select a proper signal acquisition algorithm. Similar to many engineering design problems, the selection is a trade-off between performance and cost. Since there is always a certain probability that signal acquisition is falsely declared or incorrectly missed, the performance of acquisition algorithms is expressed in terms of detection and false detection probabilities.

Serial acquisition of the GPS signal (Ward 1996) is similar to the generic direct-sequence spread spectrum (DSSS) CDMA acquisition algorithm (Peterson et al. 1995). It defines acquisition based on the comparison of signal energy against a selected threshold, which can be determined using a pre-set false detection probability, P_{FD}. The detection probability, P_D, can then be calculated from the selected threshold. This method is likely used in many existing GPS receivers. However, the serial method does not provide P_D and P_{FD} from the acquisition process. Generally it requires a priori knowledge or estimation of the carrier-to-noise ratio (C/N_0) and noise distribution in order to compute P_D and P_{FD}. The noise distribution can change with changes in the operational environment and receiver processing, and C/N_0 is usually not known to the acquisition process. As a result, even though signal acquisition may have been declared, the receiver still has limited information on the acquisition performance.

Batch processing has been widely accepted as a powerful and efficient acquisition method for GPS signals (Van

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Nee and Coenen 1991; Akos 1997; Uijt de Haag 1999; Tsui 2000), and can be easily realized in modern GPS receivers (Gunawardena et al. 2004). It calculates the circular correlation function of a batch of signal, often implemented with the fast Fourier transform (FFT).

A batch processing-based GPS signal acquisition method is used in this paper. It applies the *M*-of-*N* search algorithm (Ward 1996) to the code phase measurements obtained using the batch processing technique (Van Graas et al. 2005). Based on the repeatability of successive code phase measurements, it measures the average single trial detection probability P_d directly, and consequently the overall detection probability P_D. Its acquisition threshold is selected according to the *M*-of-*N* search algorithm, and is not calculated from C/N_0 or noise distribution. In fact, this method provides an estimate of C/N_0 based on the measured P_d in combination with the receiver processing, given the model for the noise distribution. As a batch processing-based method, multiple code phases can be searched in parallel using the same data set, which results in much less sensitivity to non-stationary noise statistics compared to serial acquisition methods.

The proposed acquisition method has been combined with various signal demodulation and integration techniques. Fundamental receiver processing techniques are used in this paper as examples, including coherent and non-coherent demodulation as well as single batch acquisition and integration over multiple single batches. The performance of the different processing techniques is evaluated using both the theoretically-derived P_d, as well as the P_d obtained from Monte–Carlo simulations.

The following section describes the code phase repeatability-based method as applied to GPS signal acquisition. Next, the mathematical relationship between acquisition performance and C/N_0 for the proposed method in combination with basic demodulation/integration techniques is presented. This is followed by a section on acquisition performance based on theoretical derivations and Monte–Carlo simulations. The last section contains the conclusions.

GPS signal detection

Signal detection based on an energy threshold

DSSS signals can be detected through the process of signal despreading followed by signal integration. If the integrated signal energy exceeds a pre-defined threshold, signal detection is declared (Peterson et al. 1995). A false detection occurs if the signal is not present where the signal energy exceeds the threshold. The selection of the detection threshold is a trade-off between probability of detection (P_d) and probability of false detection (P_{fd}). Often, the detection threshold is set to meet a particular P_{fd}. The threshold is either calculated based on known C/N_0 and noise distribution, or it is determined from a Monte–Carlo simulation (Lin and Tsui 2000). Similarly, the probability of detection, P_d, can either be calculated or obtained from a simulation. Figure 1 shows the diagram of signal detection based on an energy threshold. Also shown in this diagram is the search algorithm, since GPS signal detection requires the selection of the correct frequency and code phase for each satellite.

The energy threshold-based signal detection method requires knowledge of C/N_0 and noise distribution in order to determine both the detection threshold and the resulting P_d. An accurate value for the C/N_0 is usually not available before the signal has been acquired, since most C/N_0 estimators are implemented together with the signal tracking loops. The true noise distribution can also be difficult to determine in real-time applications. The noise distribution is affected by environmental factors, such as wide-band interference, and by receiver processing, such as demodulation and integration techniques. These factors make it difficult to accurately estimate P_{fd} and P_d.

In addition, if a serial search method is used, only one code phase and frequency offset can be evaluated at a time. With 1023 possible C/A code chips combined with 20–40 frequency search steps, it may take a generic low-end GPS receiver several seconds to complete the successful detection for a satellite with a good C/N_0 (above 45 dB-Hz). For weaker signals, the detection time is even longer. This makes the serial search method sensitive to non-stationary noise and signal strength fluctuations. The detection statistics from different code phase and frequency offsets would be obtained under different noise and/or signal strength conditions. This is particularly the case for urban and indoor positioning, where noise and signal strength can vary rapidly over several seconds. Although signal detection may still be possible, the P_d and P_{fd} estimates are no longer accurate.

Signal detection based on the repeatability of successive code phase measurements

Instead of using signal energy, GPS satellite signal detection can be based on the code phase measurement itself.

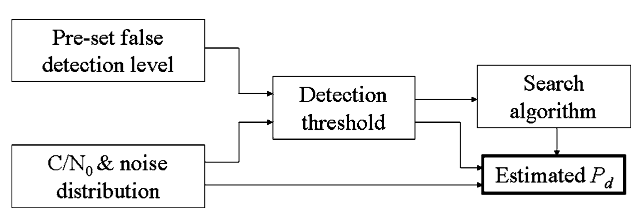

Fig. 1 Energy threshold based P_d estimation

The GPS code phase can be estimated with batch processing, such that successively independent estimates are obtained. For each estimate, or each single trial, the code phase that results in the largest correlation value is selected from a full code search (all 1023 C/A code chip offsets). The GPS code phase varies relatively slowly and smoothly over time periods of a few seconds in duration. Therefore, consecutive measurements of code phase are expected to be approximately the same in a short time window. In practice, all these measurements may not be identical due to an unknown level of noise. Nevertheless, the most repeated value of successive measurements is expected to be the true code phase, which can be identified using a histogram. In the presence of signal, the measurements that agree with the most repeated value are called ''normal operations'', whereas those different ones are not. The repeatability of the code phase measurements from N single trial batch detections estimates the probability of detection, P_d, as shown in Fig. 2.

For example, assume that the maximum relative speed between a user and a satellite is 1,000 m/s, which approximately corresponds to a shift of 3.4 chips in the phase of the C/A code during one second (the C/A code chipping rate is 1.023 MHz). If the code phase is estimated from a sequence of 300 1-ms batches, the maximum code phase change due to the relative speed is approximately 1 C/A code chip. Therefore, the tolerance threshold for normal operations can be set to ±0.5 chips for $N = 300$. Any estimate that is different from the most repeated value by more than 0.5 chips is not regarded as a normal operation.

To illustrate the estimation of detection performance, 300 code phase estimates are used. First, consider the case of a strong signal where the maximum correlation values for each 1-ms batch are found in the following locations:

estimates 1–100: 10th chip
estimate 101: 14th chip
estimates 102–300: 10th chip

The most repeated value within the 300-ms observation window is the 10th chip. All estimates other than measurement 101 are normal operations; measurement 101 is

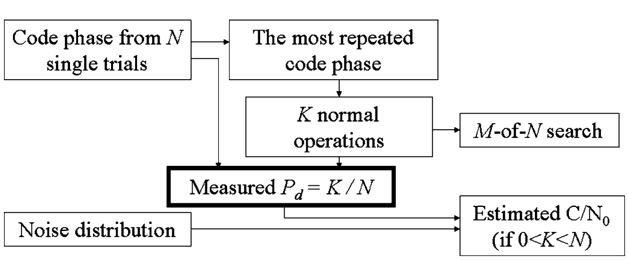

Fig. 2 Code phase repeatability-based P_d estimation

not since it has a 4-chip difference. The window size can be larger than 300 ms if the change of the code phase due to relative motion is removed using a linear fit.

In general, if K out of N estimates are normal operations, the average detection probability of a single trial can be estimated as $P_d = K / N$, as shown in Fig. 2. In the example above, $K = 299$ and $P_d = 99.67\%$. Note that the calculation of P_d does not involve the C/N_0 or the noise distribution. The P_d estimation involves N code phase measurements that are independent. Since the measurement window size is on the order of just a few 100 ms using batch processing, it can be further assumed that the noise characteristics are stationary during this time. According to the central limit theorem, the average single trial detection probability P_d has an approximately normal distribution, regardless of the noise distribution of each individual single trial detection. Therefore, the confidence interval of P_d can be determined using a normal distribution. Furthermore, based on a known noise model, the relationship between C/N_0 and P_d can be determined. When $K < N$, C/N_0 can be found from the receiver-measured P_d. When $K = N$, a minimum C/N_0 for the given signal is determined.

Next, consider 300 inputs from a weak signal, where only 80 correlation values are located at the 10th chip and the others are randomly distributed over the remaining 1,022 chips. A histogram is used to determine the most repeated code phase value. The code phase estimates that agree with the most repeated value are considered single trial detections. Most often, the performance of single trial acquisition does not satisfy the detection and false detection requirements, especially for weak signals. Search algorithms can be applied to increase the detection probability and decrease the false detection probability. For the method in this paper, the M-of-N search algorithm (Ward 1996) was adopted. The definition of the M-of-N search algorithm is adjusted for batch acquisition as follows: signal acquisition is declared if at least M-out-of-N successive code phase estimates are normal operations.

Using the proposed acquisition method and the M-of-N search algorithm, the overall detection probability P_D and its corresponding misdetection probability P_M can be derived using the single trial detection probability P_d. An overall misdetection will occur if fewer than M-out-of-N single trials are normal operations, in the presence of a signal, i.e. $K < M$. Therefore, the overall P_M is given by:

$$P_M = \sum_{i=1}^{M-1} \binom{N}{i} (1 - P_d)^{N-i} (P_d)^i, \quad P_D = 1 - P_M \tag{1}$$

A false detection represents the case that the most repeated value of the code phase estimates is caused by noise. It may happen with or without the signal, and has slightly different meanings in both cases. In the absence of signal, an overall false detection occurs when M or more out of N single trials are normal operations, where each of these M measurements is a single trial false detection. Let L stand for the number of chips per batch. It is reasonable to assume that the code phase estimate caused by noise alone is uniformly distributed over the range from the first to the Lth chip. In this case, the single trial false detection probability can be estimated as:

$$P_{\mathrm{fd},1} = 1/L. \tag{2}$$

The definition of false detection also includes the situation when there is a signal, but it is acquired with the wrong code phase. Affected by a relatively large level of noise, the most repeated value of code phase measurements detected using this method could be different from true code phase of the signal. It happens when the most repeated value is due to M or more incorrect measurements that coincidentally agree with each other. These measurements would be recognized as normal operations by mistake, and cause an ''incorrect acquisition'' of the existing signal. Since a similar effect will cause a false detection using the traditional acquisition method with serial code phase search, an incorrect acquisition of signal is also categorized as an over-all false detection event in this paper. Consequently, each of these M normal operations is a single trial false detection. Therefore, a single-trial false detection can occur in the presence of signal only if the following two conditions both hold. First, it must be an incorrect measurement, or misdetection, of the existing signal. Second, this measurement must be identical to the most repeated value, which results from M or more misdetections that coincidentally agree with each other. The single trial false detection probability P_{fd} in the presence of signal can be calculated as the product of the probabilities of both conditions. The probability of the first condition is estimated using $P_m = 1 - P_{\mathrm{d}}$ by definition. Since a misdetection of code phase has a uniform distribution over the L chips except for the true code phase, the probability for an arbitrary misdetection to agree with the most repeated value is always $1/(L - 1)$, which is the occurrence probability for the second condition. As a result, with the existence of a signal the single-trial false detection probability can be calculated using

$$P_{\mathrm{fd},2} = P_m/(L-1). \tag{3}$$

Although P_m is a function of signal strength and other parameters, as to be discussed in the following section, Eq. 3 can be over-bounded by $1/(L - 1)$ regardless. The choice of using either Eq. 2 or 3 in P_{fd} estimation is based upon whether the signal is present, which can not be determined before signal acquisition. Nevertheless, a uniform upper bound for the single trial false detection probability can be applied with or without the signal:

$$P_{\mathrm{fd}} < 1/(L-1). \tag{4}$$

This upper bound of P_{fd} is not a function of signal or noise level.

Using the M-of-N search method an overall false detection occurs if $K \geq M$, and all K measurements are due to noise. The overall P_{FD} can thus be conservatively estimated using $P_{\mathrm{fd}} = 1/(L - 1)$:

$$P_{\mathrm{FD}} = \sum_{i=M}^{N} \binom{N}{i} (1 - P_{\mathrm{fd}})^{N-i} (P_{\mathrm{fd}})^{i}. \tag{5}$$

Both Eqs. 1 and 5 have been derived from the binomial distribution (Papoulis 1965). These equations are slightly different from the original versions reported in (Ward 1996) due to the modification of the definition of the M-of-N search algorithm. In order to achieve a preset false detection performance, M can be selected for a given number of N using Eq. 5. In C/A code acquisition, P_{FD} is usually negligible even with a relatively small M, since P_{fd} has a very limited value with $L = 1{,}023$. For example, to achieve $P_{\mathrm{FD}} < 10^{-9}$ with $N = 300$, M needs to be at least 8. The overall detection probability can be estimated using Eq. 1. For the previous example, which has $K = 80$ and $P_{\mathrm{d}} = 26.7\%$, the overall detection probability becomes virtually 100%, as can be estimated using Eq. 1. Even with only nine normal operations, i.e. $K = 9$ and $P_{\mathrm{d}} = 3\%$, which has just passed the criterion of $M = 8$, the overall detection probability P_{D} becomes 55%. It has been significantly improved from the single trial P_{d}, and maintains a negligible P_{FD}. In the M-of-N search algorithm, M plays the role of an acquisition threshold in the search algorithm, which is different from an energy threshold; its value does not depend on C/N_0. For GPS C/A code, when the detection probability is below 3%, the signal C/N_0 is below 38 dB-Hz. Using code phase repeatability and the M-of-N search method, such a weak signal can be detected in 0.3 s with 1 ms integration time and a very low false detection level. Note that due to the limitation in acquisition speed, the serial process-based M-of-N search algorithm usually has small values for M and N (Ward 1996). Thus, it would be difficult to achieve high performance within a tolerable waiting time. Batch processing effectively searches L code phases within one calculation of the correlation function (Van Nee and Coenen 1991). Although the search could also be implemented with 1,023 parallel correlators, one batch FFT is more efficient in terms of the number of operations required.

In summary, the proposed acquisition method achieves detection based on the measured code phase instead of signal energy. It obtains code phase estimates from batch processing, checks the repeatability of the measurements, and finally acquires the signal using the M-of-N search algorithm. Compared to the traditional serial acquisition method, this method has the following features:

1. The signal is acquired with a known detection probability. The detection probability estimation is independent of C/N$_0$ and noise distribution, which is also approximately true for the false detection probability.
2. The acquisition threshold is not a function of C/N$_0$ or noise distribution.

Although GPS signal acquisition consists of a two-dimensional search of code phase and received carrier frequency, the carrier frequency search is omitted to simplify the analysis. The carrier frequency search can easily be added by repeating the acquisition process for different carrier frequencies. If the distribution of the noise is known, C/N$_0$ can be estimated from the receiver-measured P_d in companion with the acquisition process. The next two sections detail these relationships for both coherent and non-coherent signal processing.

Signal detection performance evaluation

Signal detection for coherent demodulation and integration

This section presents the methodology for the evaluation of the relationship between P_d and C/N$_0$ using the proposed acquisition method. Denote $R[m]$ as the correlation function of the reference pseudo random noise (PRN) sequence with the received noisy GPS signal. $0 \leq m \leq L-1$, where L stands for the batch size. Due to the characteristics of the PRN sequence, each sample of $R[m]$ can be considered to be an independent random variable. Without losing generality, assume that $R[0]$ is the correlation peak. The signal can be correctly detected if $R[0]$ has the largest amplitude of all $R[m]$. In order to estimate P_d, the cumulative probability density function (CDF) of $R[0]$, denoted by $P_1(x)$, is needed. By definition, the CDF of $R[0]$ is given by:

$$P_1(x) = P(R[0]|R[0] \leq x) \tag{6}$$

and the CDF of $R[m]$ (for any $m \neq 0$):

$$P_3(x) = P(R[m]|R[m] \leq x) \tag{7}$$

Both $P_1(x)$ and $P_3(x)$ are functions of the noise distribution. Suppose that $R[m']$ is found as the largest sample of $R[m]$ for all $m \neq 0$. The probability of $R[m']$ being no greater than x, is equivalent to the probability that all $R[m]$ values are less than or equal to x, for all $m \neq 0$, which means that

$$P(R[m']|R[m'] \leq x) = \prod_{1 \leq m \leq L-1} P(R[m]|R[m] \leq x) \tag{8}$$

The CDF of $R[m']$ can be defined as:

$$P_2(x) = P(R[m']|R[m'] \leq x) \tag{9}$$

Therefore, for any x, the CDF of $R[m']$ is given by:

$$P_2(x) = (P_3(x))^{L-1} \tag{10}$$

The probability density function (PDF) of P_2 can be found using:

$$p_2(x) = \frac{dP_2(x)}{dx} \tag{11}$$

P_m can be obtained using a conditional probability calculation as follows:

$$P_m = P(R[0]|R[0] < R[m']) = \int_{-\infty}^{+\infty} P_1(x)p_2(x)\mathrm{d}x \tag{12}$$

which, combined with the definition of P_2 in Eq. 10 and p_2 in Eq. 11, can be written as:

$$P_\mathrm{d} = 1 - P_m = 1 - (L-1)\int_{-\infty}^{+\infty} P_1(x)P_3^{L-2}(x)\frac{\mathrm{d}P_3(x)}{\mathrm{d}x}\mathrm{d}x \tag{13}$$

Derived in Eq. 13, the expression for P_d may be used to estimate probability of detection given the known noise characteristics that are needed to evaluate P_1 and P_3 from Eqs. 6 and 7, respectively. The detector noise distribution is a function of the signal demodulation and integration methodology, as explained in the following sections. The GPS signal can be demodulated either coherently or non-coherently. Both demodulation schemes, together with integration techniques, are used as examples of the proposed acquisition method.

Coherent and non-coherent demodulation

After down conversion to intermediate frequency ω, the GPS signal from a single satellite is simplified as follows:

$$s' = \sqrt{2}A \cdot \mathrm{PRN} \cdot \sin(\omega t + \varphi) + n' \tag{14}$$

where $\sqrt{2}A$ is the signal amplitude and ϕ is the carrier phase. PRN stands for the C/A code signal component with unity amplitude and the noise n' is assumed to be Additive

White Gaussian Noise (AWGN). Note that the noise power spectrum may be frequency-selective in practice, due to cross correlation, filtering and other reasons. Navigation data bits, multipath and other practical considerations have not been included in this model.

C/N_0 can be used to quantify the relative strength of DSSS signals: $C/N_0 = \text{SNR} \times \text{BW}$, (Braasch and Van Dierendonck 1999). BW is the pre-correlation bandwidth, and SNR stands for the ratio of signal power over the noise power. Based on the variance of the noise, SNR and C/N_0 can be calculated as follows (Ward 1996; Braasch and Van Dierendonck 1999):

$$\text{SNR} = \frac{A^2}{2\sigma_{n'}^2} \qquad C/N_0 = \frac{A^2 \cdot \text{BW}}{2\sigma_{n'}^2} \tag{15}$$

Thus, the noise variance can be represented as a function of C/N_0:

$$\sigma_{n'}^2 = \frac{A^2 \text{BW}}{2C/N_0} \tag{16}$$

In order to perform coherent demodulation, the frequency and phase of the carrier must be determined. Offset in demodulation frequency relative to the carrier frequency causes loss of detection energy. Therefore, a frequency search is needed to acquire the signal for most applications (Spilker 1978). Similarly, a phase offset also causes loss of detection energy. A refined search of carrier frequency and phase can be used to minimize the loss of detection energy (Soloviev et al. 2004). When the carrier frequency and the phase are precisely determined, the received signal can be demodulated as follows:

$$\begin{aligned} s_c &= s' \cdot \sqrt{2}\sin(\omega t + \varphi) = 2A \cdot \text{PRN} \\ &\quad \cdot \sin^2(\omega t + \varphi) + n' \cdot \sqrt{2}\sin(\omega t + \varphi) \\ &= A \cdot \text{PRN} \cdot (1 - \cos(2\omega t + 2\varphi)) + \text{noise}_c \end{aligned} \tag{17}$$

where noise_c represents the base band noise in coherent demodulation. The base band noise retains the same variance as $n', \sigma_c^2 = \frac{\text{BW}}{2C/N_0}$, with a normalized amplitude $A = 1$. Following the removal of the double carrier frequency component, the demodulated signal is given by:

$$s_c = A \cdot \text{PRN} + \text{noise}_c \tag{18}$$

Without searching for carrier phase, non-coherent demodulation can be used to acquire carrier frequency and code phase. The signal defined in Eq. 14 can be demodulated in two orthogonal channels:

$$\begin{aligned} s_{nc} &= s' \cdot \sqrt{2}\sin(\omega t) + js' \cdot \sqrt{2}\cos(\omega t) \\ &= 2A \cdot \text{PRN} \cdot \sin(\omega t + \varphi)\sin(\omega t) + n' \cdot \sqrt{2}\sin(\omega t) \\ &\quad + j \cdot 2A \cdot \text{PRN} \cdot \sin(\omega t + \varphi)\cos(\omega t) + jn' \cdot \sqrt{2}\cos(\omega t) \\ &= A \cdot \text{PRN} \cdot ((\cos(\varphi) - \cos(2\omega t + \varphi)) + j(\sin(\varphi) \\ &\quad + \sin(2\omega t + \varphi))) + n' \cdot \sqrt{2}e^{j\omega t} \end{aligned} \tag{19}$$

Following the removal of the double frequency components, a simplified expression of the non-coherently demodulated signal is given by:

$$s_{nc} = A \cdot \text{PRN} \cdot e^{j\varphi} + n' \cdot \sqrt{2}e^{j\omega t} \tag{20}$$

In this case, the base band noise is $\text{noise}_{nc} = n' \cdot \sqrt{2}e^{j\omega t}$. The variance of noise_{nc} is $\sigma_{nc}^2 = \frac{\text{BW}}{C/N_0}$, which is twice the noise variance of coherent demodulation, since noise now comes from two orthogonal channels. The actual performance difference between coherent and non-coherent demodulation will be shown in Sect. ''Theoretical and simulated detection performance''.

Coherent and non-coherent integrations can be performed on the signals presented in Eqs. 18 and 20, respectively. Although the focus of this paper is on C/A code processing, the same techniques can also be applied to other PRN codes, such as the P code. For example, in case of the P code, circular correlation is not used due to the long duration of the code. However, batch processing can be applied for P-code acquisition with some special techniques (Pang et al. 2003). Therefore, the detection method introduced in this paper is also applicable to P-code acquisition using batch processing.

Signal detection performance in coherent demodulation and integration

Correlation function in coherent demodulation

The correlation function for coherent acquisition is computed as follows:

$$R_c[m] = s_c[n] \otimes \text{PRN}'[n] \tag{21}$$

where $s_c\ [n]$ is the digitized signal from Eq. 18, which consists of a signal part PRN and a noise part noise_c. PRN' is the receiver-generated reference sequence, and $\otimes$ represents circular correlation. Theoretically, circular correlation is performed as:

$$\begin{aligned} R_c[m] &= \sum_{n=0}^{L-1} (\text{PRN}'[\text{mod}(n+m, L)]s_c[n]) \\ &= \sum_{n=0}^{L-1} (\text{PRN}'[\text{mod}(n+m, L)]\text{PRN}[n]) \\ &\quad + \sum_{n=0}^{L-1} (\text{PRN}'[\text{mod}(n+m, L)]\text{noise}_c[n]) \end{aligned} \tag{22}$$

The correlation function can be represented as two components; the cross correlation of PRN and PRN', denoted as R_{PP}, and the correlation of PRN' with noise$_c$, denoted as $R_{Pn,c}$. Here $R_{Pn,c}$ is referred to as correlation sample noise. These two parts are independent and can be modeled separately. Their statistical distributions will be obtained and combined to form P_1 and P_3 as introduced in Sect. ''Signal detection performance evaluation''.

First, $R_{Pn,c}$ is a linear function of noise$_c$:

$$R_{Pn,c} = \text{noise}'_c[m] = \sum_{n=0}^{L-1} (PRN'[\text{mod}(n+m, L)]\text{noise}_c[n]) \quad (23)$$

The pre-correlation noise at each sample of the signal, noise$_c$ $[n]$, is taken as independent AWGN. It then follows that each correlation sample noise noise$'_c$ $[m]$ is a Gaussian random process independent with respect to m, provided that PRN is a true random process. This property still holds approximately if PRN is pseudo random. The expected value and the variance of the correlation sample noise are given by:

$$\text{E}(\text{noise}'_c[m]) = 0, \text{VAR}(\text{noise}'_c[m]) = L\sigma_c^2 \quad (24)$$

Second, R_{PP} from Eq. 22 can be written as:

$$R_{PP} = \sum_{n=0}^{L-1} (\text{PRN}'[\text{mod}(n+m, L)]\text{PRN}[n]) \quad (25)$$

Once the satellite has been correctly detected, PRN' is a replica of PRN, such that PRN = PRN' and Eq. 25 turns into the auto correlation of PRN. The auto correlation function consists of a peak and side lobes.

$$R_{PP} = \sum_{n=0}^{L-1} (\text{PRN}[\text{mod}(n+m, L)]\text{PRN}[n]) = \begin{cases} L; m = 0 \\ \text{sidelobes}; m \neq 0 \end{cases} \quad (26)$$

The GPS C/A code auto correlation side lobes are always the same for a particular satellite, while P code side lobes only repeat after one week. For both codes, the side lobes are predictable and could be calculated and removed from Eq. 26. However, for weaker signals (C/N$_0$ below 45 dB-Hz) the side lobe levels are much smaller than the noise level in Eq. 24, such that the side lobes can be neglected.

$R_{Pn,c}$ and R_{PP} have now been modeled as two independent Gaussian random processes. The summation of them, R_c $[m]$, is also a Gaussian random process, with the mean and variance shown below.

- correlation peak $R[0]$ has $E(R[0]) = L$ and VAR$(R[0]) = L\sigma^2{}_c$;
- correlation sample noise $R[m]$, $m \neq 0$ has $E(R[m]) = 0$ and VAR$(R[m]) = L\sigma^2{}_c$.

Based on the above, $P_{1,c}$ (CDF of the peak sample, see Eq. 6) and $P_{3,c}$ (CDF of the sample noise, see Eq. 7) of $R[m]$ are obtained for coherent demodulation as follows:

$$P_{1,c}(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi L\sigma_c^2}} e^{-\frac{(y-L)^2}{2L\sigma_c^2}} \text{d}y \quad (27)$$

$$P_{3,c}(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi L\sigma_c^2}} e^{-\frac{y^2}{2L\sigma_c^2}} \text{d}y \quad (28)$$

The detection probability for coherent integration is now given by, see Eq. 13:

$$P_{\text{d,c}} = 1 - P_{m,c} = 1 - (L-1) \int_{-\infty}^{+\infty} P_{1,c}(x) P_{3,c}^{L-2}(x) \frac{\text{d}P_{3,c}(x)}{\text{d}x} \text{d}x \quad (29)$$

The detection probability is a function of $\sigma^2{}_c$. Since $\sigma_c^2 = \frac{BW}{2\text{C}/\text{N}_0}$ for coherent acquisition, $P_{\text{d,c}}$ is a function of C/N$_0$ only.

Coherent integration

Coherent integration is used to demodulate the signal coherently over longer time intervals and to accumulate the signal energy by batch addition. It has been used to acquire weak signals affected by strong in-band interference (Soloviev et al. 2004). Coherent integration for the P-code involves more complicated synchronization problems (Pang et al. 2003) and is beyond the scope of this paper. Assuming that the data bit transitions are known, the energy of N batches of continuous signal can be accumulated using the summation of correlation functions:

$$R_{c,i}[m] = \sum_{j=0}^{N-1} (s_{c,j}[n] \otimes \text{PRN}'_j[n]) \quad (30)$$

where $s_{c,0}$ through $s_{c,N-1}$ represent N batches of the GPS signal. Since the C/A code is periodic, PRN$'_j$ always equals PRN and Eq. 30 is equivalent to:

$$R_{c,i}[m] = \left(\sum_{j=0}^{N-1} s_{c,j}[n]\right) \otimes \text{PRN}[n] \quad (31)$$

Similar to the analysis in Sect. ''Correlation function in coherent demodulation'', the expected value and variance of the correlation peak and sample noise can be found. E(R[0]) = NL, and VAR(R[0]) = NLσ_c^2. For any $m \neq 0$, E($R[m]$) = 0 and VAR($R[m]$) = NLσ_c^2.

The detection probability for coherent integration over N batches is obtained from Eq. 13 as follows:

$$P_{d,c,i} = 1 - P_{m,c,i} = 1 - (L-1)\int_{-\infty}^{+\infty} P_{1,c,i}(x)P_{3,c,i}^{L-2}(x)\frac{\mathrm{d}P_{3,c,i}(x)}{\mathrm{d}x}\mathrm{d}x \quad (32)$$

where

$$P_{1,c,i}(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi L\sigma^2}} e^{-\frac{(y-L)^2}{2L\sigma^2}}\mathrm{d}y \quad (33)$$

and

$$P_{3,c,i}(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi L\sigma^2}} e^{-\frac{y^2}{2L\sigma^2}}\mathrm{d}y \quad (34)$$

with $\sigma^2 = \frac{\mathrm{BW}}{2N\cdot \mathrm{C/N_0}}$. Thus, the detection probability for coherent integration is a function of C/N$_0$ and the integration length N. Compared to the single batch coherent acquisition, N-batch coherent integration reduces the noise level by a factor of N, which equals $10\log_{10} N$ in dB.

Signal detection in non-coherent demodulation and integration

Correlation function in non-coherent demodulation

The correlation function for non-coherent demodulation can be written as:

$$R_{nc}[m] = |s_{nc}[n] \otimes \mathrm{PRN}'[n]| \quad (35)$$

where s_{nc} [n] is the digitized non-coherent signal from Eq. 20 and '| x |' means taking the magnitude of x. Similar to the coherent demodulation in Eq. 22, Eq. 35 can be represented as the summation of two components.

$$R_{nc}[m] = |R_{PP}[m] + R_{Pn,nc}[m]| = |\mathrm{PRN}[n]e^{j\varphi} \otimes \mathrm{PRN}[n] + \mathrm{noise}_{nc} \otimes \mathrm{PRN}[n]| \quad (36)$$

where R_{PP} is the cross correlation of the received PRN sequence and the reference PRN sequence, and $R_{Pn,nc}$ is the non-coherent correlation sample noise.

Note that $\mathrm{noise}_{nc} = n' \cdot \sqrt{2}e^{j\omega t}$, as shown in Eq. 20. Due to the random phase of noise$_{\mathrm{nc}}$, the correlation peak R[0] has a Rayleigh distribution and the off-peak $R[m]$ has a Ricean distribution (Ward 1996). The PDF of $R[m]$, $m \neq 0$ is given by:

$$p_3(z) = \frac{z}{\sigma^2}e^{-\left(\frac{z^2}{2\sigma^2}\right)} \quad (37)$$

where $\sigma^2 = L\sigma_n^{2\prime}$. As given by Eq. 16, $\sigma_{n'}^2 = \frac{\mathrm{BW}}{2\mathrm{C/N_0}}$. Thus, Eq. 37 can be written as:

$$p_3(z) = \frac{z}{L\sigma_{n'}^2}e^{-\left(\frac{z^2}{2L\sigma_{n'}^2}\right)} \quad (38)$$

The CDF can be computed as

$$P_{3,nc}(x) = \int_{-\infty}^{x} p_3(z)\mathrm{d}z \quad (39)$$

The PDF of R[0] is given by:

$$p_1(z) = \frac{z}{L\sigma_{n'}^2}e^{-\left(\frac{z^2+L^2}{2L\sigma_{n'}^2}\right)} I_0\left(\frac{zL}{L\sigma_{n'}^2}\right) \quad (40)$$

Likewise, its CDF can be computed as:

$$P_{1,nc}(x) = \int_{-\infty}^{x} p_1(z)\mathrm{d}z \quad (41)$$

To calculate the detection probability for non-coherent acquisition, Eqs. 39 and 41 are substituted into 13.

Non-coherent integration

Similar to coherent integration, signal energy can be accumulated to improve the non-coherent acquisition performance. Non-coherent integration is a useful method for weak signal acquisition, since neither an intensive search of carrier frequency nor navigation data bit information is required. However, the benefit in terms of P_{d} provided by non-coherent integration is not as large as that for coherent integration. This section evaluates the effect of non-coherent integration on P_{d} and C/N$_0$.

Although a coherently integrated correlation function has the same Gaussian distribution as a single batch correlation, only with different mean and noise variance, non-coherent integration results in a different post-correlation noise distribution. Therefore, P_1 and P_3 for non-coherent are different than for coherent integration. Non-coherent integration can be written as:

$$R_{nc,i}[m] = \sum_{j=0}^{N-1} R_{nc,j}[m] = \sum_{j=0}^{N-1} \left| s_{nc,j}[n] \otimes \mathrm{PRN}'_j[n] \right| \quad (42)$$

which is the summation of magnitudes from N single batch correlations. The PDF of $R[0]$ is obtained as the convolution of the PDFs of all $R_{nc,j}$ [0] involved (Papoulis 1965):

$$p_{1,nc,i} = \mathrm{conv}(\mathrm{conv}(..(\mathrm{conv}(p_{nc0,0}, p_{nc0,1}), p_{nc0,2})\ldots), p_{nc0,N-1}) \quad (43)$$

where $p_{nc0,i}$ represents the PDF of $R_{nc,j}$ [0]. Although it is difficult to find a closed form solution for Eq. 43, the numerical calculation of $p_{1,nc,i}$ can be implemented. $P_{1,nc,i}$ is obtained using Eq. 41. Similar to Eq. 43, p_3 is determined in a similar way:

$$p_{3,nc,i} = \mathrm{conv}(\mathrm{conv}(..(\mathrm{conv}(p_{ncm,0}, p_{ncm,1}), p_{ncm,2})\ldots), p_{ncm,N-1}) \quad (44)$$

where $p_{ncm,j}$ represents the PDF of $R_{nc,j}$ $[m]$, $m \neq 0$. $P_{3,nc,i}$ is obtained using Eq. 39. After P_1 and P_3 are determined numerically, Eq. 13 is used to calculate the detection probability for non-coherent integration. Again, this probability is a function of C/N_0 and the integration time interval.

Theoretical and simulated detection performance

In this section, the code phase repeatability-based method and the P_d analysis are applied to coherent and non-coherent GPS C/A code acquisition techniques. The proposed method is a valuable tool for performance evaluation of different GPS receiver designs. In order to verify the theoretical values for P_d, Monte–Carlo computer simulations are performed for different receiver implementations.

The simulation of base-band single satellite acquisition was performed at a sampling rate of 1.023 Msps, the C/A code chip rate. The simulation was repeated 10,000 times for single 1-ms batch acquisition and 10-batch integration acquisition to determine P_d. Figure 3 shows the theoretical and simulated P_d as a function of C/N_0 using coherent and non-coherent single-batch acquisition. The lines represent the theoretical values, while the circles and stars represent the simulated results. Also shown with the simulated results are the corresponding 99% confidence intervals. As can be seen from Fig. 3, the theoretical values for the detection probabilities agree well with the Monte–Carlo simulation results. For the C/N_0 shown in Fig. 3, coherent acquisition outperforms non-coherent acquisition by 1–2 dB.

As described earlier in this paper, the traditional method of signal detection compares the correlation level for each single trial, which is often from the non-coherent approach, against a pre-selected threshold decided based on a desired false detection level (Ward 1996; Lin and Tsui 2000). Since the proposed acquisition method is based on a different principle of signal detection, it is useful to compare the detection performance of these two methods, which can be measured with the single trial detection probability P_d. Note that the use of the proposed acquisition method results in a fixed upper bound of the single trial false detection probability $P_{fd} < 1/(L - 1)$, which is used to set the detection threshold for the traditional method.

As for regular GPS signal acquisition, non-coherent demodulation is used for the comparison. The statistical distribution of the correlation peak and sample noise from single batch correlation can be found from Eqs. 39 and 41, which, in turn, are used to calculate the detection threshold for a range of C/N_0 levels. A 10,000-sample Monte–Carlo simulation is performed at each C/N_0 level, and P_d is estimated using both the threshold-based method and the code phase repeatability-based method proposed in this paper, as shown in Fig. 4. P_d from the threshold-based method, shown as circles, and P_d from the proposed method, shown as a line, demonstrate a clear similarity between the detection performance of the two methods. The difference between the two methods is that, for the proposed method, the actual detection probability is known, which is not the case for the threshold-based method, unless knowledge of the C/N_0 is provided.

Single batch acquisition may not be able to detect weak signals and integration over time can be used to increase the detection probability. Due to potential navigation data

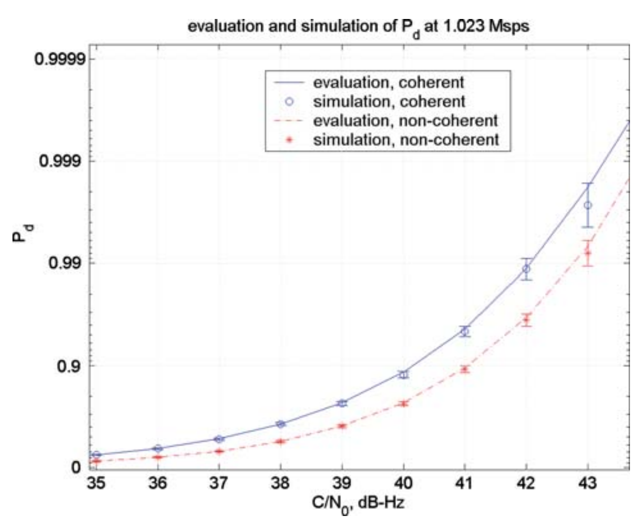

Fig. 3 Theoretical and simulated P_d using single-batch coherent and non-coherent demodulation

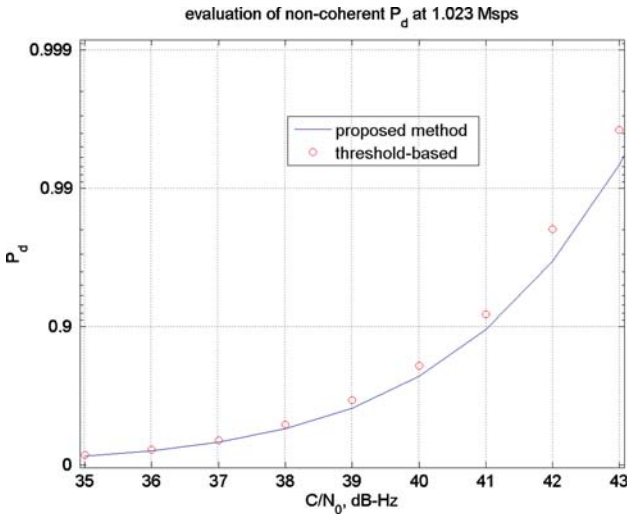

Fig. 4 P_d for non-coherent detection, with code phase repeatability-based detection and the threshold-based detection

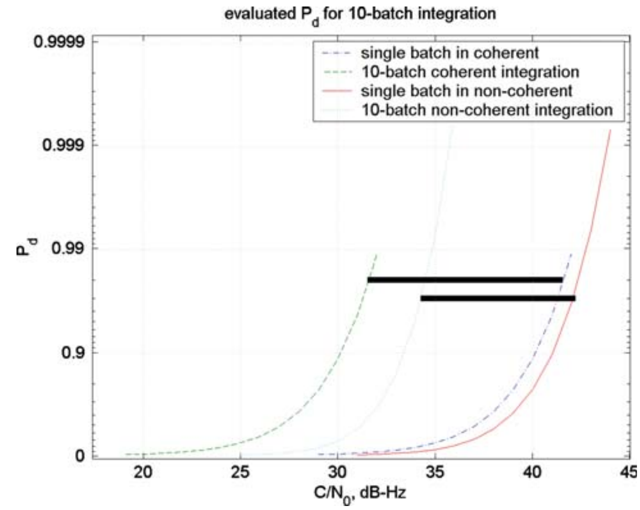

Fig. 6 Theoretical P_d for coherent and non-coherent detection using single-batch and 10-batch processing

bit transitions every 20 ms, a reasonable integration size is ten 1-ms batches (Tsui 2000). Figure 5 shows the theoretical and simulated P_d as a function of C/N$_0$ using 10-batch integrated coherent and non-coherent acquisition. The theoretical P_d values using integration are further compared against the results from single-batch acquisitions in Fig. 6. Figure 6 shows a 10 dB gain through 10-batch coherent integration, shown as the longer horizontal line. For non-coherent integration, the gain is approximately 8 dB as indicated by the shorter horizontal line in Fig. 6.

In general, N-batch coherent integration provides for $10\log_{10} N$ dB gain for the same detection probability, while the gain using non-coherent integration is smaller. The longer the integration length, the larger the C/N$_0$ loss for

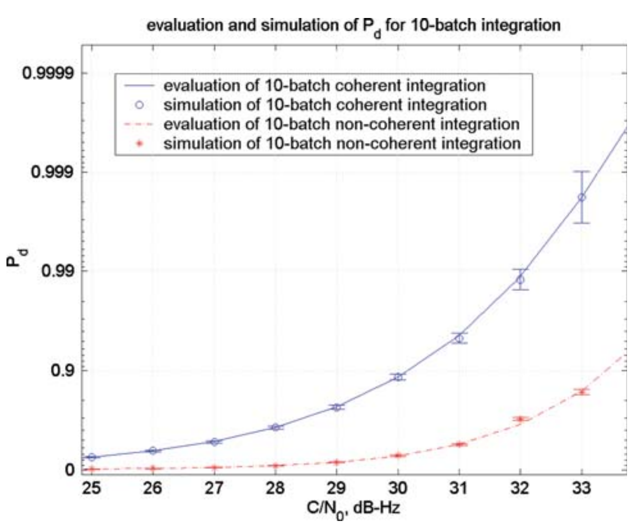

Fig. 5 Theoretical and simulated P_d for coherent and non-coherent detection using 10-batch integration

non-coherent integration relative to coherent integration. Note, however, that the improvement for non-coherent integration is still significant at 8 dB for $N = 10$. Considering the more demanding frequency search required for coherent integration, non-coherent integration may be a practical option for many applications.

It is noted that the above results are also valid for higher sampling rates (e.g. 5 Msps) since the noise level of DSSS signals is measured using the power spectral density N$_0$, which is not affected by the sampling rate.

In an actual receiver implementation, P_d is obtained using the repeatability of the code phase measurements. Therefore, C/N$_0$ can be estimated using a pre-generated look-up table of C/N$_0$ with respect to P_d. For example, Fig. 3 can be used to produce a look-up table based on P_d measured from single batch coherent or non-coherent acquisition. It can be applied to signals with C/N$_0$ ranging from 35 to 43 dB-Hz. A table generated from Fig. 4 would be based on P_d for 10-batch integrations covering 25–34 dB-Hz. Additional look-up tables can be generated using the equations provided in Sect. ''Signal detection performance evaluation''. This particular C/N$_0$ estimator can be integrated as a part of the signal detection process with no need for signal tracking, which is a desirable feature for weak signal acquisition.

Conclusions

A signal detection method based on the repeatability of successive code phase measurements is introduced for batch GPS signal acquisition. The acquisition performance, including detection and false detection probabilities is estimated during signal acquisition, without knowledge of

C/N_0 or noise distribution. If the distribution of the noise is known, the code phase repeatability-based method also provides an estimate of C/N_0. This method has been successfully applied to coherent and non-coherent GPS signal acquisition.

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