EE 313: Basic EE
Chapter 5 Synopsis

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AC Circuits

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Motivation

AC **is the backbone of the electric power industry—also, about 80% of EE 315 (energy conversion) employs AC circuit analysis**

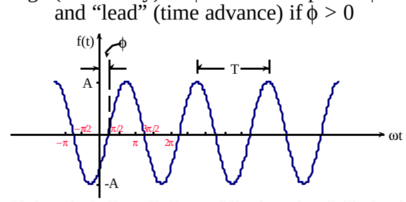
Frequency response **of electric circuits (to AC signals) is the conceptual "key" to understanding** ***filters*** **(tuners, graphic equalizers, etc.) found in consumer electronics, etc.**

5.1 Sinusoidal (AC) Sources

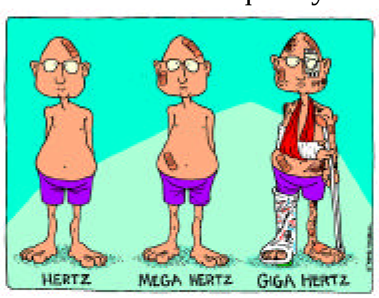
Generic Sinusoid: $f(t) = A\sin(\omega t + \phi)$

where A = Amplitude, ω = Radian Frequency (Rad./Sec.), f = Frequency (Hz., formerly cps) where $\omega = 2\pi f$, $T = 1/f$ = Period (in Sec., mSec., etc.) and

ϕ = Phase angle (in Rad. or Degrees) where f(t) is said to "Lag" (time delay) its $\phi = 0$ counterpart if $\phi < 0$ and "lead" (time advance) if $\phi > 0$



Remember: ω = *radian frequency* is in rad./sec. whereas f = frequency is in Hz.



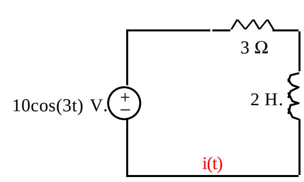
5.2 Steady-State Response of Circuits to AC Sources

Below is a simple example of an AC circuit analysis problem.

Note that the circuit is driven by an (ideal) AC voltage source

The presumed *objective* is to find the *steady-state current* i(t)

N.B.: Problems of this ilk are endemic to (inter alia) the electric power engineering field.



So how's it done?

5.2 Traditional Differential Equations Approach

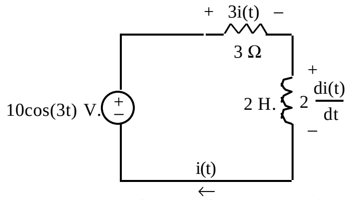
KVL for the RL circuit below yields the differential equation:

$2(di(t)/dt) + 3i(t) = 10\cos(3t)$

The *mathematical* objective is to find a *particular solution* of the equation (i.e., *any* solution that will satisfy the equation)

The standard approach is to choose

$i(t) = I_C\cos 3t + I_S\sin 3t$ (an educated guess, but why?)



5.2 Traditional Differential Equations Approach

Substituting $i(t) = I_C\cos 3t + I_S\sin 3t$ into the differential equation $2(di(t)/dt) + 3i(t) = 10\cos(3t)$ yields:

$2(-3I_C\sin 3t + 3I_S\cos 3t) + 3(I_C\cos 3t + I_S\sin 3t) = 10\cos(3t)$ or …

$(3I_C + 6I_S)\cos 3t + (-6I_C + 3I_S)\sin 3t = 10\cdot\cos 3t + 0\cdot\sin 3t$

Equating coefficients yields:

$3I_C + 6I_S = 10$ and $-6I_C + 3I_S = 0$

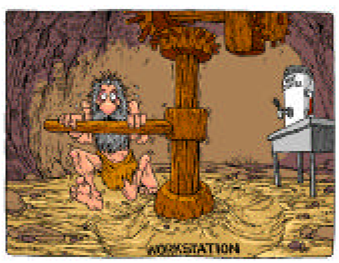
Which have solution $I_C \approx 0.667$ and $I_S \approx 1.33$

So a particular solution is $i(t) \approx 0.667\cdot\cos 3t + 1.33\cdot\sin 3t$

However, $A\cdot\cos(\omega t) + B\cdot\sin(\omega t) = (A^2 + B^2)^{1/2}\cos(\omega t - \tan^{-1}(B/A))$

$\therefore i(t) \approx 1.491\cos(3t - 63.43°)$

There's got to be a better way!



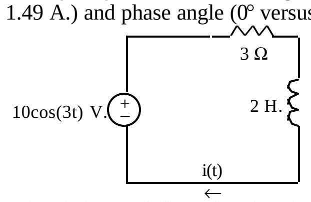
And there is, thanks to Charles Proteus Steinmetz (1865-1923)

5.2 Traditional Differential Equations Approach

However, before proceeding, note that the circuit's current response to the sinusoidal voltage source is $i(t) \approx 1.49\cos(3t - 63.43°)$ A.

How do the voltage source and current response differ mathematically?

They differ by only two *numbers*—magnitude (10 V. versus 1.49 A.) and phase angle (0° versus –63.43°)!



5.3 Complex Numbers and Complex Arithmetic

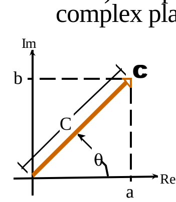
The *phasor method* (invented in 1893 by Charles Proteus Steinmetz, 1865-1923) is the backbone of AC steady-state circuit analysis

The principle benefit of the phasor method is that its concepts essentially render AC steady-state circuit analysis analogous to DC steady-state analysis—by way of complex arithmetic —so that the *entire* differential-equations process is circumvented!

However, in order to gainfully employ the phasor method, the electric circuit analyst must be proficient in working with complex numbers, complex arithmetic and complex algebra

5.3 Complex Numbers and Complex Arithmetic cont.

A complex number manifests itself in two forms—and consists of two numbers (per form) and presents itself as a vector in the complex plane as shown below



$\mathbf{C} = a + jb$ (Rectangular form)

$\mathbf{C} = C\,e^{j\theta} = C\,/\underline{\theta}$ (Polar form)

where

$j = (-1)^{1/2}$, $a = \mathrm{Re}(\mathbf{C}) = C\cos\theta$, $b = \mathrm{Im}(\mathbf{C}) = C\sin\theta$

$C = (a^2 + b^2)^{1/2}$ and $\theta = \tan^{-1}(b/a)$

5.3 Complex Numbers and Complex Arithmetic cont.

Addition and subtraction of complex numbers is defined in rectangular form *only* as follows:

If **A** = a + jb and **B** = c + jd then

A + **B** = (a + c) + j(b + d) and

A – **B** = (a – c) + j(b – d)

Multiplication and division is defined in polar form as follows:

If **A** = $A\angle\theta_A$ and **B** = $B\angle\theta_B$ then

AB = $AB\angle\theta_A + \theta_B$ and

A/B = $A/B\angle\theta_A - \theta_B$

and in rectangular form as follows ...

5.3 Complex Numbers and Complex Arithmetic cont.

Multiplication and division in rectangular form:

If **A** = a + jb and **B** = c + jd then

AB = (a + jb)(c + jd)

= (ac – bd) + j(ad + bc) and

A/B = (a + jb)/(c + jd)

= $[(ac + bd) + j(bc - ad)]/(c^2 + d^2)$

Also, the *conjugate* of a phasor **A** = a + jb = $C\angle\theta$ is

$\mathbf{A}^*$ = a – jb = $C\angle{-\theta}$ so that $\mathbf{AA}^*$ = (a + jb)(a – jb)

= $a^2 + b^2 = C^2$ also …

1 = +1 + j0 = $1\angle 0$; –1 = –1 + j0 = $1\angle\pm 180°$

and j = 0 + j1 = $1\angle 90°$

5.3 Complex Numbers and Complex Arithmetic cont.

Given: **A** = 1 – j3 and **B** = $2\angle{-30°}$, the following complex arithmetic yields the following numerical results (see text Problem 5.7, p. 202):

A + **B** = (1 – j3) + $2\angle{-30°}$ ≈ (1 – j3) + (1.732 – j1)

≈ 2.732 – j4 ≈ $4.844\angle{-55.67°}$

A – **B** = (1 – j3) – $2\angle{-30°}$ ≈ (1 – j3) – (1.732 – j1)

≈ –0.732 – j2 ≈ $2.130\angle{-110.1°}$

AB = (1 – j3)($2\angle{-30°}$) ≈ ($3.162\angle{-71.57°}$)($2\angle{-30°}$)

≈ $6.324\angle{-101.57°}$ ≈ –1.268 – j6.195

A/B ≈ ($3.162\angle{-71.57°}$)/($2\angle{-30°}$) ≈ $1.581\angle{-41.57°}$

≈ 1.183 – j1.049

1/**A** ≈ $1\angle 0°$/($3.162\angle{-71.57°}$) ≈ $0.3163\angle{+71.57°}$

≈ 0.1 – j0.3 and $\mathbf{AA}^* \approx 3.162^2 \approx 10.0$

5.4 Use of Complex Exponential Source

The complex-exponential-source technique is the *conceptual bridge* to the forthcoming phasor technique (which is ubiquitous in practice)

In AC steady-state circuit analysis, sinusoidal sources are mathematically related to complex exponential sources via Euler's Equation: $e^{jx} = \cos(x) + j\sin(x)$

(Euler's Equation provides the *mathematical bridge*)

Any particular solution for exponential source(s) is also a complex exponential, and is *much* easier to determine than its sinusoidal counterpart—in fact, it can be found using (complex) arithmetic alone!

Finally, the particular (trigonometric) solution for *any* AC circuit can be readily deduced from its complex exponential counterpart

5.4 Use of Complex Exponential Source cont.

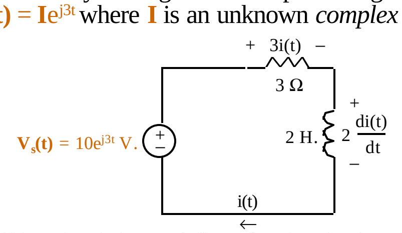
Recapitulating, the exponential-source technique provides the *bridge* to the *phasor method*

To see this, consider the circuit analyzed earlier (using differential equations)—but now driven instead by the exponential counterpart of its original sinusoidal source, namely ...

5.4 Use of Complex Exponential Source cont.

The differential equation is: 2(di(t)/dt) + 3i(t) = $V_s(t)$

Let the particular solution *resemble* the source so that it too is a complex exponential differing from the source only in magnitude and phase angle; i.e., let **i(t) = I**e^{j3t} where **I** is an unknown *complex number*



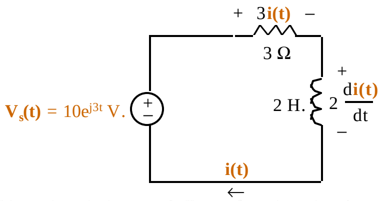
5.4 Use of Complex Exponential Source cont.

Then $2(d\mathbf{i(t)}/dt) + 3\mathbf{i(t)} = \mathbf{V_s(t)}$ or ...

$2(d(\mathbf{I}e^{j3t})/dt) + 3(\mathbf{I}e^{j3t}) = 10e^{j3t}$ V. or…

$2(j3)\mathbf{I}e^{j3t} + 3(\mathbf{I}e^{j3t}) = 10e^{j3t}$ V. or …

$(3 + j6)\mathbf{I}e^{j3t} = 10e^{j3t}$ V. and the e^{j3t} factors (time functions) cancel leaving …

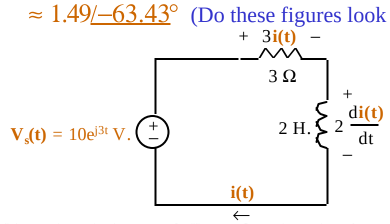


5.4 Use of Complex Exponential Source cont.

$\mathbf{I} = 10/(3 + j6)$—which is an equation in complex numbers *only* (no time functions at all) which yields the magnitude and phase angle unique to the current response, namely …

$\mathbf{I} = 10\underline{/0}°/(3 + j6) \approx (10\underline{/0}°)/(6.71\underline{/63.43}°)$

$\approx 1.49\underline{/-63.43}°$ (Do these figures look familiar?)



5.4 Use of Complex Exponential Source cont.

Therefore, $\mathbf{i(t)} = \mathbf{I}e^{j3t} \approx 1.49\underline{/-63.43}°e^{j3t}$ A.

$\approx 1.49e^{-j63.43°}e^{j3t}$ A. $\approx 1.49e^{j(3t-63.43°)}$ A.

and …

If $V_s(t) = 10\cos(3t)$ V. = Re[$10e^{j3t}$ V.], what's i(t)?

$i(t) \approx \mathrm{Re}[1.49e^{j(3t-63.43°)}] \approx 1.49\cos(3t - 63.43°)$ A.

Likewise

If $V_s(t) = 10\sin(3t)$ V. = Im[$10e^{j3t}$ V.], what's i(t)?

$i(t) \approx \mathrm{Im}[1.49e^{j(3t-63.43°)}] \approx 1.49\sin(3t - 63.43°)$ A.

The final task is to develop a method to streamline the calculations even further

This method is ...

5.5 The Phasor Circuit (Method)

Recall that , $\mathbf{I} = 10/(3 + j6)$ which is an equation in complex numbers (no time functions at all!)

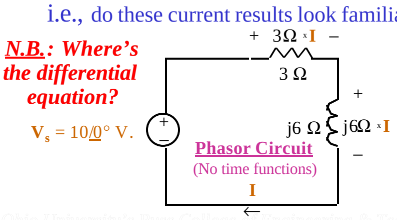
How does this compare with the calculation of

$\mathbf{I} = 10\underline{/0}°/(3 + j6)\ \Omega \approx (10\underline{/0}°\text{ V.})/(6.71\underline{/63.43}°\ \Omega)$

$\approx 1.49\underline{/-63.43}°$A. for the *conceptual* circuit below;

i.e., do these current results look familiar?

***N.B.*: Where's the differential equation?**



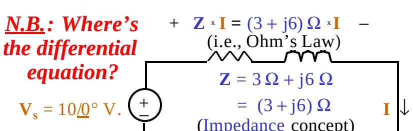
5.5 The Phasor Circuit cont.

The phasor circuit can be redrawn as shown below

Note that the phasor circuit provides a *(conceptual)* *schematic tool* for finding the magnitude and phase angle of the current (which are its only unique properties!)

Of the complex *numbers* $\mathbf{V_s}$, $\mathbf{Z}$ and $\mathbf{I}$, which do not correspond to time functions?

***N.B.*: Where's the differential equation?**



By KVL, $\mathbf{I} = \mathbf{V_s} / \mathbf{Z} = (10\underline{/0}°\text{ V.}) / (3 + j6)\Omega \approx 1.49\underline{/-63.43}°$A.

5.5 The Phasor Circuit cont.

Only currents and voltages are phasors

(i.e., have time-function counterparts)

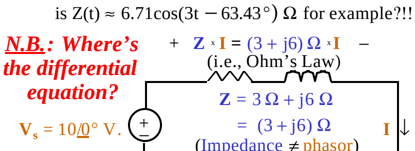
Don't believe it? Then what's Z(t) for the phasor circuit below if the phasors correspond to cosinusoids

(so that $V_s(t) = 10\cos(3t)$ V. and $i(t) \approx 1.49\cos(3t - 63.43°)$ A.)?

Since $\mathbf{Z} = (3 + j6)\ \Omega \approx 6.71\underline{/63.43}°\ \Omega$,

is $Z(t) \approx 6.71\cos(3t - 63.43°)\ \Omega$ for example?!!

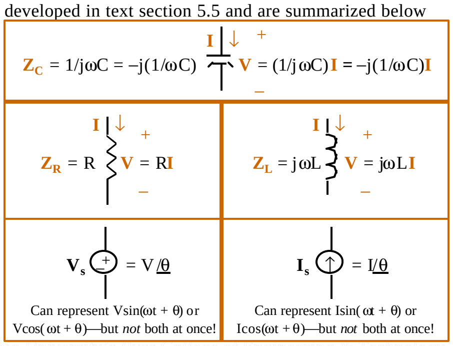
***N.B.*: Where's the differential equation?**



By KVL, $\mathbf{I} = \mathbf{V_s} / \mathbf{Z} = (10\underline{/0}°\text{ V.}) / (3 + j6)\Omega \approx 1.49\underline{/-63.43}°$A.

5.5 The Phasor Circuit cont.

The "Rules and Regulations" for phasor sources and impedances are developed in text section 5.5 and are summarized below



5.5 The Phasor Circuit cont.

The following equations from text p.171 are useful in reconciling situations where both sinusoidal and cosinusoidal sources are present

$$\sin(\omega t + \theta) = \cos(\omega t + \theta - 90°)$$

$$\cos(\omega t + \theta) = \sin(\omega t + \theta + 90°)$$

(see example below)

If Vcos(ωt + θ)'s phasor representation is ⇒ V /θ

Then Esin(ωt + ϕ)'s phasor representation is ⇒ E /ϕ − 90°

5.5 The Phasor Circuit cont.—Example 5-1

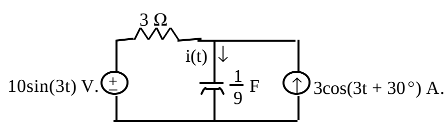
Find the AC steady-state current i(t)—see text problem 5.26 (p.204)

Let the phasors represent cosinusoids so that the current source's phasor current is 3/30° A.

Then the voltage source's phasor voltage is 10/−90° V.

The capacitor's impedance is $-j\{1/[(3 \text{ Rad./sec.})(1/9 \text{ F})]\} = -j3\ \Omega$

So the phasor circuit becomes ...



5.5 The Phasor Circuit cont.—Example 5-1 cont.

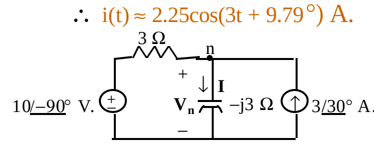
KCL at node n:

$(10\underline{/-90°}\text{ V.} - \mathbf{V_n}) / 3\Omega - \mathbf{V_n} / (-j3\ \Omega) + 3\underline{/30°}\text{ A.} = 0\text{ A.}$

Which has solution $\mathbf{V_n} \approx 6.745\underline{/-80.21°}$ V.

Then $\mathbf{I} = \mathbf{V_n} / (-j3\ \Omega) \approx 2.25\underline{/+9.79°}$ A.

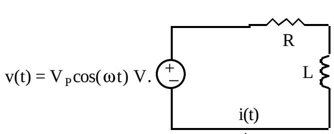
∴ i(t) ≈ 2.25cos(3t + 9.79°) A.



Voila!

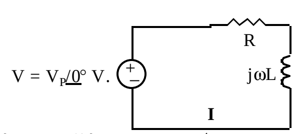
5.6 Average Power, Reactive Power and Power Factor

What's the nature of power in AC steady-state circuits? Consider the following RL circuit …
What's the circuit's *instantaneous* power P(t)?



5.6 Average Power, Reactive Power and Power Factor cont.

Applying KVL to the phasor circuit below yields the current's phasor from which i(t) is deduced
Then the *instantaneous* power is P(t) = v(t)i(t)



$$\mathbf{I} = \frac{V_P\underline{/0°}}{R + j\omega L} = \frac{V_P\underline{/0°}}{Z\underline{/\theta}} = \frac{V_P}{Z}\underline{/-\theta} = I_P\underline{/-\theta} \quad \therefore \quad i(t) = I_p\cos(\omega t - \theta)\text{ A.}$$

Where $Z = [R^2 + (\omega L)^2]^{1/2}$ and $\theta = \tan^{-1}(\omega L/R)$

5.6 Average Power, Reactive Power and Power Factor cont.

$P(t) = v(t)i(t) = V_p\cos(\omega t)I_p\cos(\omega t - \theta)$

$= 0.5V_pI_p\cos(\theta)\{1 + \cos(2\omega t)\} + 0.5V_pI_p\sin(\theta)\{-\sin(2\omega t)\}$

Recall from text p. 39 that $X_{RMS} = X_{PEAK}/\sqrt{2}$ ∴ since $2 = \sqrt{2}\sqrt{2}$,

$P(t) = V_{RMS}I_{RMS}\cos(\theta)\{1 + \cos(2\omega t)\} + V_{RMS}I_{RMS}\sin(\theta)\{-\sin(2\omega t)\}$

∴ $P(t) = Pf_P(t) + Qf_Q(t)$, where …

$P = V_{RMS}I_{RMS}\cos(\theta)$ & $Q = V_{RMS}I_{RMS}\sin(\theta)$

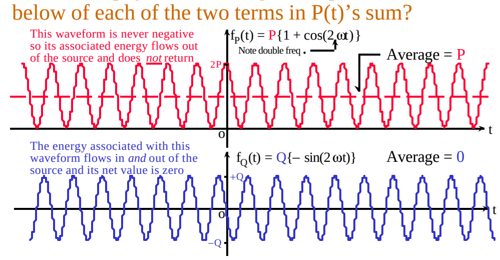
are *constants* and

$f_P(t) = \{1 + \cos(2\omega t)\}$ and $f_Q(t) = -\sin(2\omega t)$

are *time functions*

5.6 Average Power, Reactive Power and Power Factor cont.

What's the physical meaning of the plots shown below of each of the two terms in P(t)'s sum?



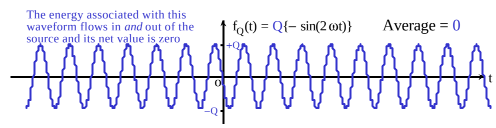
N.B.: The average of $P(t) = Pf_P(t) + Qf_Q(t)$ is P ∴ P is *average power*

Q: Remember what P is?

A: It's why Mr. DVM is upset!

N.B.: Given the CB hasn't tripped, estimate the *maximum* power (in hp) Mr. DVM is absorbing.

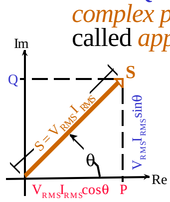
Q is "sloshing back & forth" power which occurs as a consequence of LC energy storage charging & discharging in the AC environment



P is the *business end* of AC power, whereas, although Q is a part of the AC power *package*, it's not particularly useful power (in fact, it can be a problem, e.g.; see power factor correction)

5.6 Average Power, Reactive Power and Power Factor cont.

P and Q as sides of the *Power Triangle and* P and Q as the real and imaginary parts of *complex power* **S** (whose magnitude S is called *apparent power*)



$S = P + jQ = V_{RMS}I_{RMS}\cos\theta + jV_{RMS}I_{RMS}\sin\theta$

$S = Se^{j\theta} = S/\theta = V_{RMS}I_{RMS}/\theta$

Complex power (S) is a mathematical artifact!

N.B.: θ is called the *power factor angle* where the power factor is either cos(θ) lag if θ > 0, cos(θ) lead if θ < 0 or unity (θ = 0)

5.6 Average Power, Reactive Power and Power Factor cont.

What's *complex power* **S** in terms of RMS voltage and current phasors?

What *are* RMS phasors?

An RMS phasor is it's traditional (peak) phasor counterpart divided by √2 (see text p. 39)

Then *complex power* $\mathbf{S} = \mathbf{V}_{RMS}\mathbf{I}_{RMS}^*$

Where $\mathbf{V}_{RMS} = \mathbf{V}/\sqrt{2}$ and $\mathbf{I}_{RMS} = \mathbf{I}/\sqrt{2}$

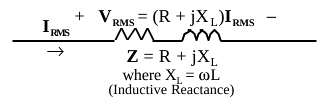
5.6 Average Power, Reactive Power and Power Factor cont.

Then since

complex power $\mathbf{S} = \mathbf{V}_{RMS}\mathbf{I}_{RMS}^*$ be sure to

Remember: Before making any power related calculations from phasors, *always* convert traditional (peak) phasors to RMS phasors!!

5.6 Average Power, Reactive Power and Power Factor— some useful formulae

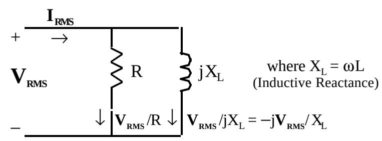


$\mathbf{S} = \mathbf{V}_{RMS}\mathbf{I}_{RMS}^* = RI^2_{RMS} + jX_L I^2_{RMS} = P + jQ_L$

$\therefore P = RI^2_{RMS}$ and $Q_L = X_L I^2_{RMS}$

(Note similarity to DC formulae)

5.6 Average Power, Reactive Power and Power Factor— some useful formulae cont.



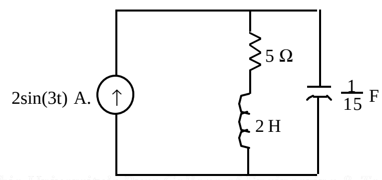
$\mathbf{S} = \mathbf{V}_{RMS}\mathbf{I}_{RMS}^* = V^2_{RMS}/R + jV^2_{RMS}/X_L = P + jQ_L$

$\therefore P = V^2_{RMS}/R$ and $Q_L = V^2_{RMS}/X_L$

(Note similarity to DC formulae)

5.6 Average Power, Reactive Power and Power Factor—Example 5-2

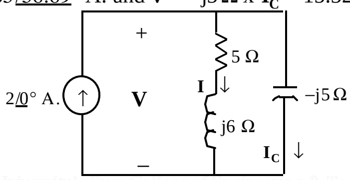
Calculate the complex power produced by the source and consumed by each passive element (see text problem 5.31, p. 205)



5.6 Average Power, Reactive Power and Power Factor—Example 5-2 cont.

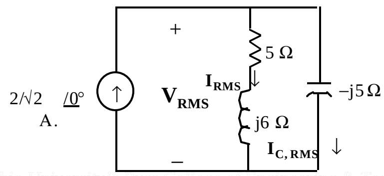
Phasor circuit: $\omega = 3$ Rad./Sec., $j\omega L = j3{\times}2 = j6\,\Omega$, $-j(1/\omega C) = -j5\,\Omega$, and the source phasor is $2\underline{/0°}$ A.

By current ÷, $\mathbf{I} = [-j5\,\Omega/(5\,\Omega + j6\,\Omega - j5\,\Omega)]2\underline{/0°}$ A. Ergo, $\mathbf{I} \approx 1.961\underline{/-101.3°}$ A. and by KCL, $\mathbf{I}_C = 2\underline{/0°}$ A. – $\mathbf{I}$ Ergo $\mathbf{I}_C \approx 3.063\underline{/38.89°}$ A. and $\mathbf{V} = -j5\,\Omega \times \mathbf{I}_C \approx 15.32\underline{/-51.1°}$ V.



5.6 Average Power, Reactive Power and Power Factor—Example 5-2 cont.

Dividing each current and voltage phasor by √2 yields the RMS phasors: $1.414\underline{/0°}$ A. (source phasor), $\mathbf{I}_{RMS} \approx 1.387\underline{/-101.3°}$ A., $\mathbf{I}_{C,RMS} \approx 2.166\underline{/38.89°}$ A. and $\mathbf{V}_{RMS} \approx 10.83\underline{/-51.1°}$ V.

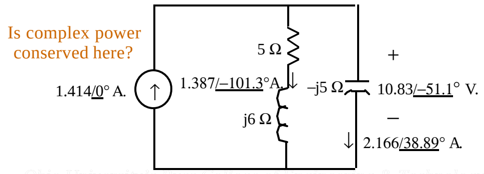


5.6 Average Power, Reactive Power and Power Factor—Example 5-2 cont.

Then the source's complex power is $\mathbf{V}_{RMS}$ (1.414/0° A.)* ≈ 15.31/–51.1° VA., ≈ 9.61W – j11.91 VAR

The complex power of the resistor, inductor and capacitor are respectively $(I_{RMS})^2R$ = (1.387 A.)²(5Ω) ≈ 9.61W, $j(I_{RMS})^2X_L$ = j(1.387 A.)²(6Ω) ≈ j11.52 VAR and $-j(V_{RMS})^2/X_C$ = –j(10.83 V.)²/5Ω ≈ –j23.46 VAR

Is complex power conserved here?



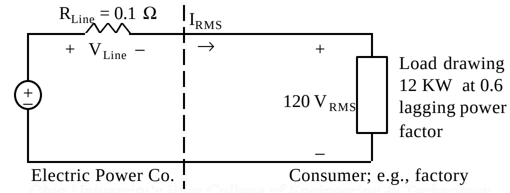
5.6 Average Power, Reactive Power and ***Power Factor Correction***: An ***Application***

S = P/cos(θ) = 12 KW/0.6 = 20 KVA = $V_{RMS} I_{RMS}$

∴ I_{RMS} = 20 KVA/120V ≈ 167 A. (RMS)

Ergo, Line losses = $I^2_{RMS} R_{Line}$ ≈ 2.78 KW (≈ 23% of load power)

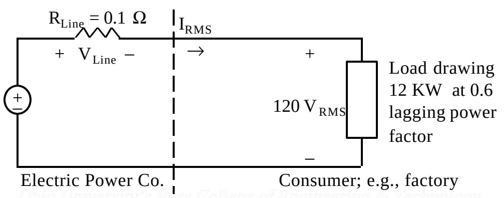
and $V_{Line} = R_{Line} I_{RMS}$ ≈ 16.7 V (≈ 14% of load voltage)



5.6 Average Power, Reactive Power and ***Power Factor Correction***: An ***Application*** cont.

The load's complex power is $\mathbf{S}_{Load}$ = P + jQ
= 12 KW + j|$\mathbf{S}_{Load}$|sin(θ) = 12 KW + j20 KVA × 0.8
= 12 KW + j16 KVAR

Next, add -j7 KVAR of capacitance in parallel with the load to obtain ...

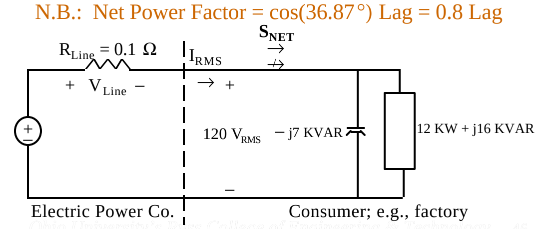


5.6 Average Power, Reactive Power and ***Power Factor Correction***: An ***Application*** cont.

The net (load and capacitor) complex power is:

$\mathbf{S}_{NET}$ = $\mathbf{S}_{Load}$ –j7 KVAR = 12 KW + j16KVAR –j7 KVAR
= 12 KW + j9 KVAR ≈ 15/36.87° KVA

N.B.: Net Power Factor = cos(36.87°) Lag = 0.8 Lag

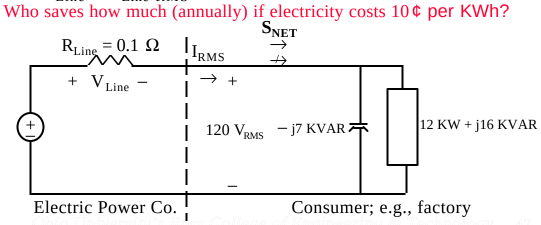


5.6 Average Power, Reactive Power and ***Power Factor Correction***: An ***Application*** cont.

Then S_{NET} = 15 KVA = $V_{RMS} I_{RMS}$ = (120 V_{RMS}) I_{RMS} yields I_{RMS} = 15 KVA/120 V. = 125 A. (RMS) Ergo, ...

Line losses = $I^2_{RMS} R_{Line}$ ≈ 1.56 KW (≈ 13% of load power)
and $V_{Line} = R_{Line} I_{RMS}$ ≈ 12.5 V (≈ 10.4% of load voltage)

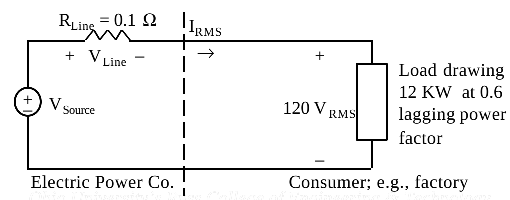
Who saves how much (annually) if electricity costs 10¢ per KWh?



5.6 Average Power, Reactive Power and ***Power Factor Correction***: An ***Application***

Example 5-3

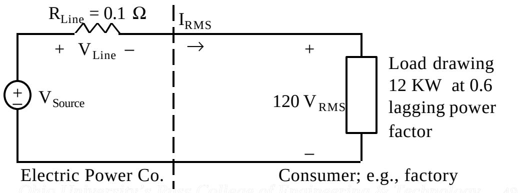
For the previous power factor correction application, what's the source's apparent power, power factor and voltage magnitude with and without the power factor correction?



5.6 Average Power, Reactive Power and ***Power Factor Correction***: An ***Application***

Example 5-3 cont.

Without correction, the source produces $\mathbf{S}_{Source} = \mathbf{S}_{Load} + \mathbf{S}_{Line}$ = (12 KW + j16 KVAR) + (2.78 KW + j0 KVAR) = 14.78 KW +j16 KVAR = 21.78$\angle$**42.27°** KVA $\therefore$ the apparent power is 21.78 KVA and the power factor is cos(42.27°) Lag = 0.74 Lag

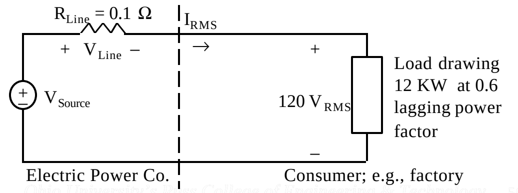


5.6 Average Power, Reactive Power and ***Power Factor Correction***: An ***Application***

Example 5-3 cont.

The current is I_{RMS} = 167 A. and the apparent power ($V_{Source} \times I_{RMS}$) is 21.78 KVA

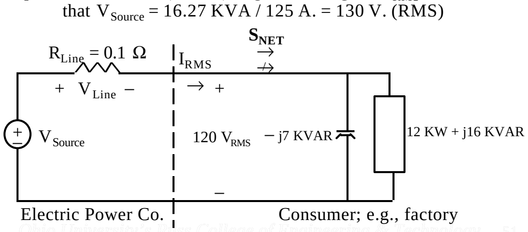
$\therefore$ V_{Source} = 21.78 KVA / 167 A. = 130 V. (RMS)



5.6 Average Power, Reactive Power and ***Power Factor Correction***: An ***Application***

Example 5-3 cont.

With correction, the source produces $\mathbf{S}_{Source} = \mathbf{S}_{Net} + \mathbf{S}_{Line}$ = (12 KW + j9 KVAR) + (1.56 KW + j0 KVAR) = 13.56 KW + j9 KVAR = 16.27$\angle$**33.57°** KVA $\therefore$ the apparent power is 16.27 KVA, the power factor is cos(33.57°) Lag = 0.83 Lag and I_{RMS} = 125 A. so that V_{Source} = 16.27 KVA / 125 A. = 130 V. (RMS)



5.6 Average Power, Reactive Power and ***Power Factor Correction***: An ***Application***

Example 5-3 cont.

Summary

PF Correction	Apparent Power[1]	Power Factor[2]	Source Voltage[3]
Without	21.78 KVA	0.74 Lag	130 V.
With	16.27 KVA	0.83 Lag	130 V.

[1] Apparent power is germane to the source's *size* (rating) and therefore its capital cost

[2] The power factor indicates the percentage of the source's *apparent* power that is *real* power (83% versus 74%) and thus is a measure of the source's efficacy

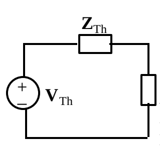
[3] Although this voltage is invariant across cases here, in practice, voltage regulation is usually improved by power-factor correction

5.6 Average Power, Reactive Power and Power Factor—***Maximum Power Transfer***

The steady-state AC version of the maximum power transfer theorem is similar to its DC counterpart (but the numbers are complex) and is shown below—see text pp. 183-185 for its (calculus) derivation

Note that both the (Thevenin) phasor circuit's voltage $\mathbf{V}_{Th}$ and impedance $\mathbf{Z}_{Th}$ are complex numbers

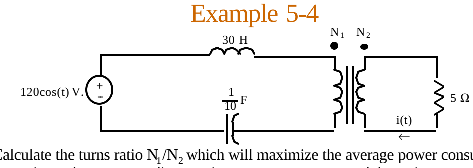
The rules for obtaining the Thevenin circuit remain: $\mathbf{V}_{Th} = \mathbf{V}_{open\ circuit}$ and $\mathbf{Z}_{Th}$ = circuit impedance remaining after all independent sources are deactivated



$\mathbf{Z}_{Load} = \mathbf{Z}^*_{Th}$ for maximum (average) power transfer from the Thevenin circuit to the load— $R_{Th} = Z_{Th}$ (magnitude of $\mathbf{Z}_{Th}$) if the load is purely resistive

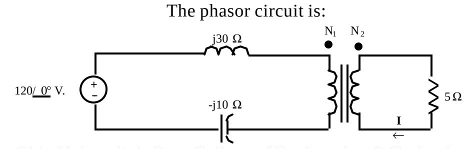
5.6 Average Power, Reactive Power and Power Factor—***Maximum Power Transfer*** cont.

Example 5-4



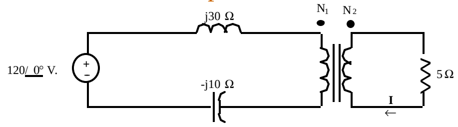
Calculate the turns ratio N_1/N_2 which will maximize the average power consumed by the resistor, the corresponding maximum average power and the resistor current i(t)

The phasor circuit is:

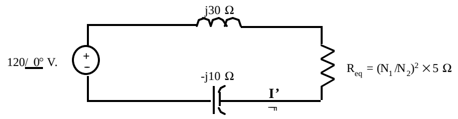


5.6 Average Power, Reactive Power and Power Factor—***Maximum Power Transfer*** cont.

Example 5-4 cont.

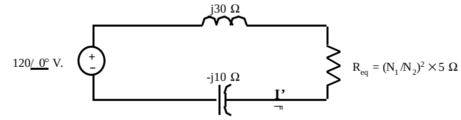


Reflecting the resistor across the transformer yields:

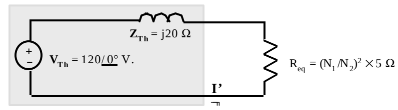


5.6 Average Power, Reactive Power and Power Factor—***Maximum Power Transfer*** cont.

Example 5-4 cont.

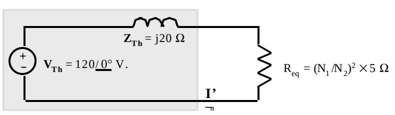


Combining the two series reactances yields the (resistance-loaded) Thevenin circuit:



5.6 Average Power, Reactive Power and Power Factor—***Maximum Power Transfer*** cont.

Example 5-4 cont.



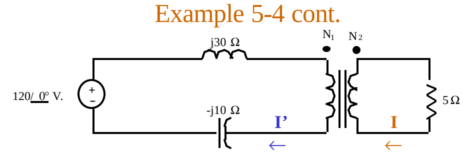
For maximum average power, $R_{eq} = (N_1/N_2)^2 \times 5\ \Omega = |\mathbf{Z_{Th}}| = 20\ \Omega$

$\therefore N_1/N_2 = 2$

Then $\mathbf{I'} = [\mathbf{V_{Th}} = 120\angle 0°\ \text{V.}] / [\mathbf{Z_{Th}} + R_{eq} = (20 + j20)\ \Omega] = 3\sqrt{2}\ \angle -45°$ A.

5.6 Average Power, Reactive Power and Power Factor—***Maximum Power Transfer*** cont.

Example 5-4 cont.



As a result of the transformer's turns ratio, $\mathbf{I} = (N_1/N_2)\ \mathbf{I'}$

$\therefore \mathbf{I} = (N_1/N_2)\ \mathbf{I'} = 2 \times 3\sqrt{2}\angle -45°$ A. $= (6\sqrt{2})\angle -45°$ A. so that

$i(t) = (6\sqrt{2})\cos(t - 45°)$ A. » $8.485\cos(t - 45°)$ A.

and $I_{RMS} = |\mathbf{I}|/\sqrt{2} = 6$ A. so that $P_{ave.,\,max.} = (I_{RMS})^2 \times 5\ \Omega$ yields:

$P_{ave.,\,max.} = 180$ W.

5.7 Frequency Response of Circuits

- Time-domain signals have a frequency-domain counterpart, e.g.; music is a time varying sound signal with, inter alia, bass, mid-range and treble characteristics (components)
- The concomitant mathematical relationships were discovered in 1822 by Jean Baptiste Joseph Fourier (1768-1830)
- See text equation 5.129 (p. 185) and its accompanying Figure 5.20 (p. 186) = next slide, as well as text Appendix C (p. 753)
- Signals of all sorts (e.g., sound) can be converted to electrical signals (voltage and current waveforms) via transducers
- Electrical signals can be "processed" by electric circuits because electric circuit behavior is frequency dependent

5.7 Frequency Response of Circuits cont.

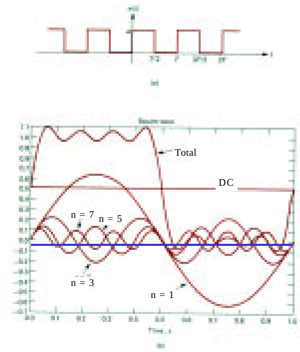
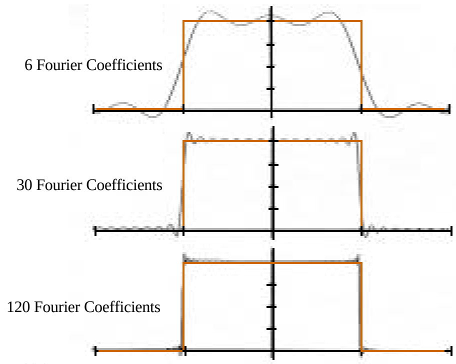


FIGURE 5.20 Contributions of Fourier series components to total waveform for square wave.

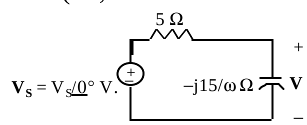
5.7 Frequency Response of Circuits cont.



5.7 Frequency Response of Circuits cont.

- Consider the circuit of text Figure P5.39 (p. 207) whose phasor circuit is shown below
- Voltage division produces the ratio (*transfer function*) **H**(jω) = **V**/$\mathbf{V}_S$ = (–j15/ω)/(5 – j15/ω) = 1/(1 + jω/3) which in this case is *gain*

(i.e., what are **H**'s units?)



5.7 Frequency Response of Circuits cont.

- This *transfer function* can be written in polar form as **H**(jω) = M(ω)$\underline{/\theta(\omega)}$ where:

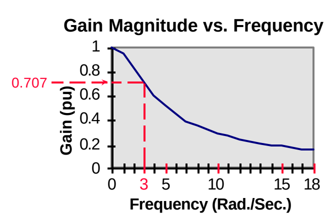
  M(ω) = {1/[1 + (ω/3)²]^½} and θ(ω) = –tan⁻¹(ω/3)
- So both the gain's magnitude and phase angle depend on frequency; i.e., M(0) = 1 (which is also M's maximum) and θ(0) = 0° whereas Lim[M(ω)] and Lim[θ(ω)] as ω → ∞ are 0 and –90° respectively
- Note also that for (ω/3) >> 1, M(ω) ≈ 3/ω (which means higher gain for lower frequencies)
- What's M(ω) and θ(ω) when ω = 3 Rad./Sec.?

5.7 Frequency Response of Circuits cont.

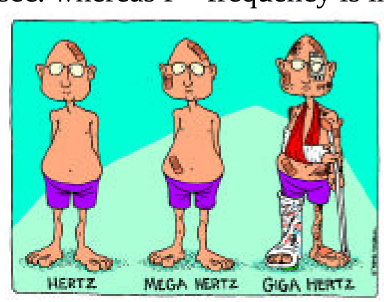
- M(3) = {1/[1 + (3/3)²]^½} = 1/√2 and θ(ω) = –45° which is the transfer function's *half-power point*
- Why the moniker *half-power point*?
- Suppose $\mathbf{V}_S = V_S\underline{/0}$° V. for all frequencies (i.e., all ω)
- Then the maximum output voltage $V_{max.} = V_S$ occurs at ω = 0 since V= M(ω)V_S and $M_{max.}$ = M(0) = 1
- Impressing this voltage across a load resistor yields the maximum output power $P_{max.} = V_S^2/R_L$
- The *output* voltage at ω = 3 Rad./Sec. is M(3)V_S = V_S /√2 and the corresponding power is $(V_S/\sqrt{2})^2/R_L$ = $[V_S^2/R_L]/2$ = $P_{max.}/2$ (*half of* $P_{max.}$)

5.7 Frequency Response of Circuits cont.

A plot of M(ω) is shown below

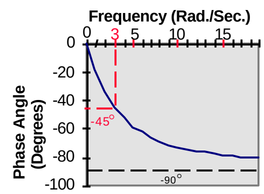


Remember: ω = *radian frequency* is in rad./sec. whereas f = frequency is in Hz.



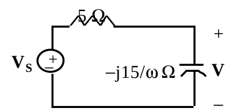
5.7 Frequency Response of Circuits cont.

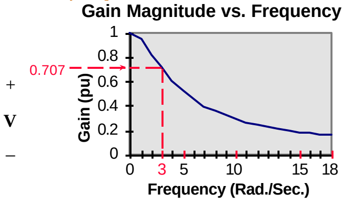
A plot of $\theta(\omega)$ is shown below



5.7.2 Filters

The RC circuit just considered is a *low-pass filter* because the magnitude of its gain $M(\omega)$ *favors lower frequencies*





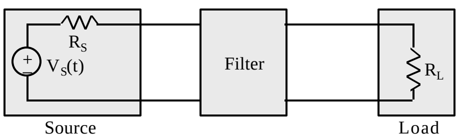
Transfer function's gain magnitude = $|\mathbf{H}(j\omega)| = |\mathbf{V}/\mathbf{V_S}| = V/V_S = M(\omega)$

N.B.: If the capacitor's voltage favors low frequencies, one can intuit that the resistor's voltage favors high frequencies (why?)

5.7.2 Filters cont.

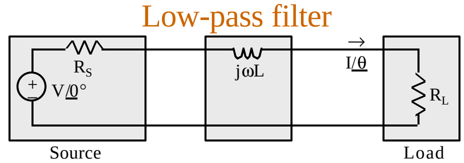
Other types of filters are possible and are developed in text section 5.7.2 (pp. 190-200) using the generic prototype shown below

The *results* for each filter are ...



5.7.2 Filters cont.

Low-pass filter



$\mathbf{H}(j\omega) = I/\underline{\theta} \,/\, V/\underline{0}° = \{ V/\underline{0}° \,/\, (R_S + R_L + j\omega L)\} / \, V/\underline{0}°$

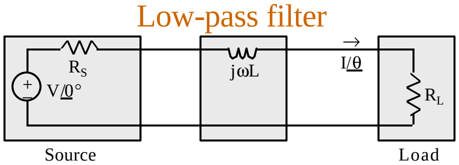
for which

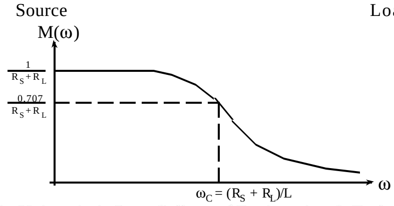
$$M(\omega) = \frac{1}{R_S + R_L} \cdot \frac{1}{(1 + [\omega L/(R_S + R_L)]^2)^{1/2}}$$

Which has a cut-off (half-power) frequency of $\omega_C = (R_S + R_L)/L$

5.7.2 Filters cont.

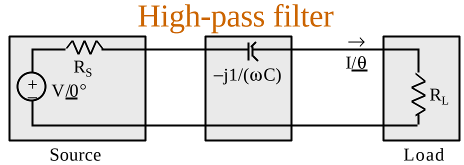
Low-pass filter





5.7.2 Filters cont.

High-pass filter



$\mathbf{H}(j\omega) = I/\underline{\theta} \,/\, V/\underline{0}° = \{ V/\underline{0}° \,/\, [R_S + R_L - j(1/\omega C)]\} / \, V/\underline{0}°$

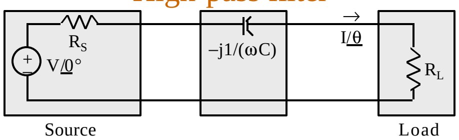
for which

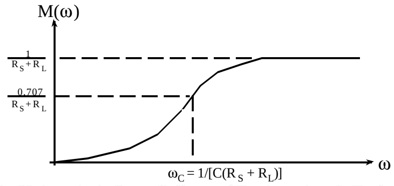
$$M(\omega) = \frac{1}{R_S + R_L} \cdot \frac{1}{\{1 + [1/(\omega C(R_S + R_L)]^2\}^{1/2}}$$

Which has a cut-off (half-power) frequency of $\omega_C = 1/[C(R_S + R_L)]$

5.7.2 Filters cont.

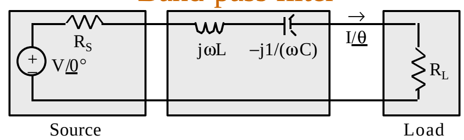
High-pass filter





5.7.2 Filters cont.

Band-pass filter



$\mathbf{H}(j\omega) = I\angle\theta \,/\, V\angle 0° = \{V\angle 0° \,/\, [R_S + R_L + j\omega L - j(1/\omega C)]\} / V\angle 0°$

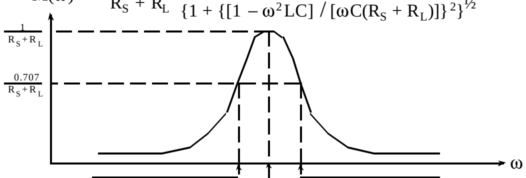
for which

$$M(\omega) = \frac{1}{R_S + R_L} \cdot \frac{1}{\{1 + \{[1 - \omega^2 LC] \,/\, [\omega C(R_S + R_L)]\}^2\}^{1/2}}$$

5.7.2 Filters cont.

Band-pass filter

$$M(\omega) = \frac{1}{R_S + R_L} \cdot \frac{1}{\{1 + \{[1 - \omega^2 LC] \,/\, [\omega C(R_S + R_L)]\}^2\}^{1/2}}$$



$$\omega_l = -\frac{R_S + R_L}{2L} + \left[\left(\frac{R_S + R_L}{2L}\right)^2\right]^{1/2}$$

(Lower half-power frequency)

$$\omega_0 = 1/(LC)^{1/2}$$

(Center frequency)

$$\omega_u = +\frac{R_S + R_L}{2L} + \left[\left(\frac{R_S + R_L}{2L}\right)^2\right]^{1/2}$$

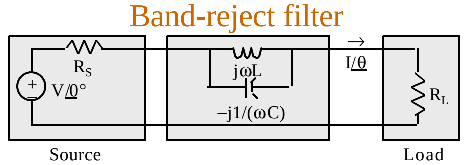
(Upper half-power frequency)

$B = \omega_u - \omega_l = (R_S + R_L)/L$ (Bandwidth)

$Q = \omega_0 / B = \omega_0 L/(R_S + R_L)$ (Selectivity)

5.7.2 Filters cont.

Band-reject filter



$\mathbf{H}(j\omega) = I\angle\theta \,/\, V\angle 0° = V\angle 0° \,/\, [R_S + R_L + (j\omega L) \,||\, (-j(1/\omega C))]$

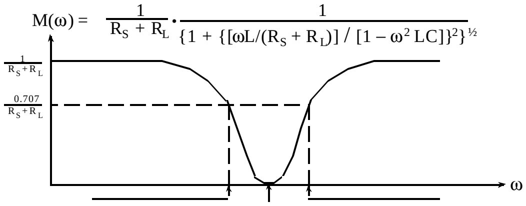
for which

$$M(\omega) = \frac{1}{R_S + R_L} \cdot \frac{1}{\{1 + \{[\omega L/(R_S + R_L)] \,/\, [1 - \omega^2 LC]\}^2\}^{1/2}}$$

5.7.2 Filters cont.

Band-reject filter

$$M(\omega) = \frac{1}{R_S + R_L} \cdot \frac{1}{\{1 + \{[\omega L/(R_S + R_L)] \,/\, [1 - \omega^2 LC]\}^2\}^{1/2}}$$



$\omega_l = -\alpha + (\alpha^2 + \omega_0^2)^{1/2}$ (Lower half-power frequency)

$\omega_0 = 1/(LC)^{1/2}$ (Center frequency)

$\omega_u = \alpha + (\alpha^2 + \omega_0^2)^{1/2}$ (Lower half-power frequency)

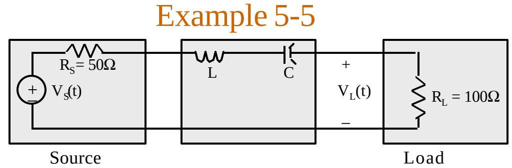
$$\alpha = \frac{1}{2C\,(R_S + R_L)}$$

$B = \omega_u - \omega_l = 1/[C(R_S + R_L)]$ (Bandwidth)

$Q = \omega_0 / B = \omega_0 C(R_S + R_L)$ (Selectivity)

5.7.2 Filters cont.

Example 5-5



Design the band-pass filter to have a center frequency of 1 MHz. and a bandwidth of 10 kHz. What's this filter's Q?

$\omega_0 = 2\pi f_0 = 2\pi\times 10^6$ Rad./Sec. $= 1/(LC)^{1/2}$ = (Center frequency)

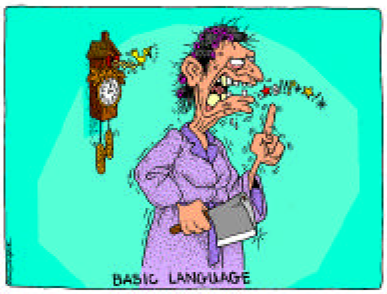
$B = 2\pi f_{BW} = 2\pi\times 10^4$ Rad./Sec. $= (R_S + R_L)/L = 150\,\Omega/L$ (Bandwidth)

$\therefore$ $L = 150\,\Omega \,/\, (2\pi\times 10^4$ Rad./Sec.$) \approx 2.39$ mH

and $C = 1 / (\omega_0^2 L) \approx 1/[(2\pi\times 10^6$ Rad./Sec.$)^2 \times 2.39$ mH$] \approx 10.6$ pF

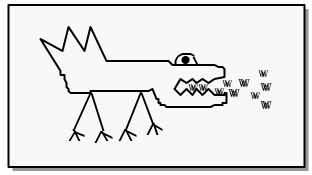
and $Q = \omega_0 / B = (2\pi\times 10^6$ Rad./Sec.$) / (2\pi\times 10^4$ Rad./Sec.$) = 100$

Recommendation: Use ***spreadsheet software*** to solve homework problem 5.47



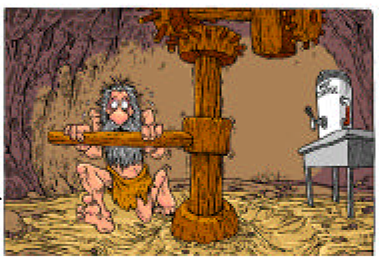
Examination

Identify the EE *instrumentation* shown below



Homework

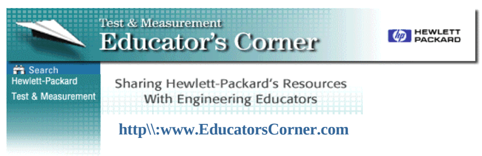
Exam Payoff



1O5 Problems ∴ ≈ 11 per week

Teamwork!

Acknowledgement:
Cartoons courtesy of



We welcome your questions with

Enthusiasm!!