EE 313: Basic EE
Chapter 3 Synopsis
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Chapter 3 Synopsis

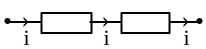
Analytic Techniques for Resistive Circuits

3.1 Series and Parallel Connections of Resistors

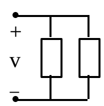
Elements in series share the same *physical** current

Elements in parallel share the same *physical** voltage

* *Physical* as opposed to *numerical*; e.g., two elements having the same numerical current are not necessarily in series, etc.



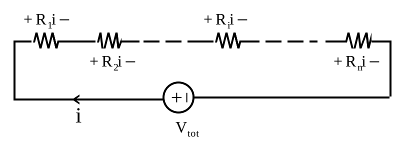
Two elements in series



Two elements in parallel

3.1 Series and Parallel Connections of Resistors

Resistors in series add: $R_{tot} = R_1 + R_2 + \ldots + R_n$



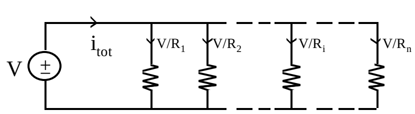
3.1 Series and Parallel Connections of Resistors

Resistors in parallel add reciprocally:

$R_{tot}^{-1} = R_1^{-1} + R_2^{-1} + \ldots + R_n^{-1}$

And conductances in parallel add directly:

$G_{tot} = G_1 + G_2 + \ldots + G_n$ (See text p. 60)

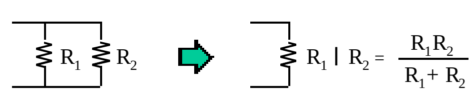


3.1 Series and Parallel Connections of Resistors

Special case for *two* Resistors in parallel:

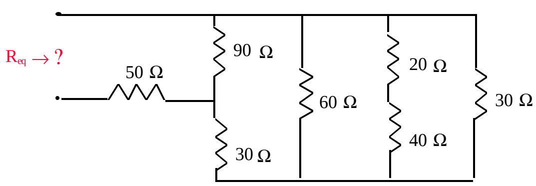
$R_1 \| R_2 = (R_1R_2)/(R_1 + R_2)$

N.B.: Product-over-sum rule and notation (see text p. 61)—memorize!



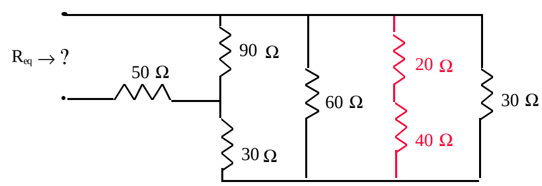
3.1 Series and Parallel Resistors: Example 3-1

Example 3-1: Calculate R_{eq}



3.1 Series and Parallel Resistors: Example 3-1

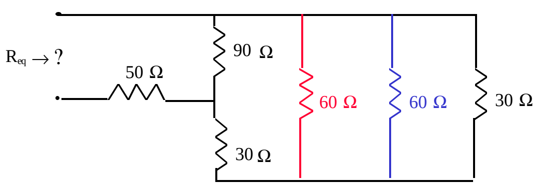
Example 3-1: Calculate R_{eq}



Step 1: 20 Ω + 40 Ω = 60 Ω (two series resistors)

3.1 Series and Parallel Resistors: Example 3-1

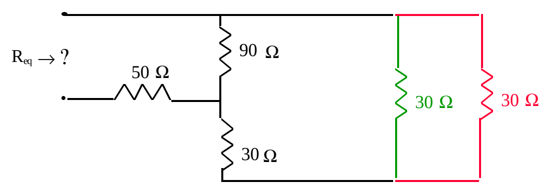
Example 3-1: Calculate R_{eq}



Step 2: 60 Ω | 60 Ω = 30 Ω (two parallel resistors)

3.1 Series and Parallel Resistors: Example 3-1

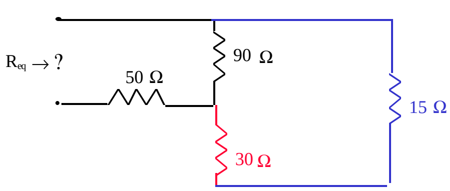
Example 3-1: Calculate R_{eq}



Step 3: 30 Ω | 30 Ω = 15 Ω (two parallel resistors)

3.1 Series and Parallel Resistors: Example 3-1

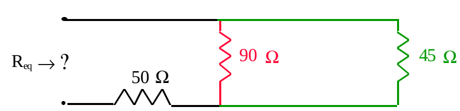
Example 3-1: Calculate R_{eq}



Step 4: 30 Ω + 15 Ω = 45 Ω (two series resistors)

3.1 Series and Parallel Resistors: Example 3-1

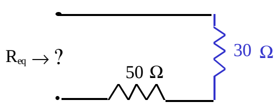
Example 3-1: Calculate R_{eq}



Step 5: 90 Ω | 45 Ω = 30 Ω (two parallel resistors)

3.1 Series and Parallel Resistors: Example 3-1

Example 3-1: Calculate R_{eq}



Step 6: 50 Ω + 30 Ω = 80 Ω (two series resistors)

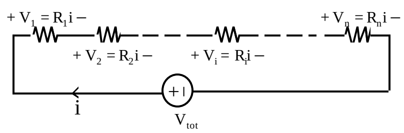
3.1 Series and Parallel Resistors: Example 3-1

Example 3-1: Calculate R_{eq}

Step 7: Ergo, R_{eq} = 80 Ω

3.2 Voltage and Current division

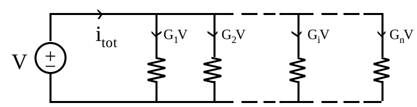
Voltage division: Series resistor voltages divide in proportion to resistances



$$\frac{V_i}{V_{tot}} = \frac{R_i}{R_{tot}}$$ *Memorize!*

3.2 Voltage and Current division

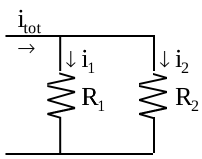
Current division: Parallel resistor currents divide in proportion to conductances



$$\frac{i_i}{i_{tot}} = \frac{G_i}{G_{tot}}$$ *Memorize!*

3.2 Voltage and Current division

Special case: Two Resistors in parallel:
$i_1/i_{tot} = R_2/(R_1 + R_2)$, etc. (see below)
N.B.: "Other" resistor over sum of resistors (see text p. 68)—memorize!



$$\frac{i_1}{i_{tot}} = \frac{R_2}{R_1 + R_2}$$

$$\frac{i_2}{i_{tot}} = \frac{R_1}{R_1 + R_2}$$

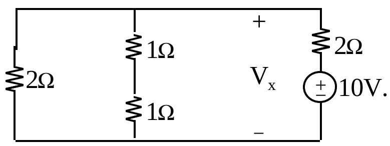
Memorize!

Circuit Analysis Tool kit: Recapitulation

- Single-loop circuit analysis
- Single-node-pair circuit analysis
- Source transformations
- Series-parallel resistance combinations
- Voltage division
- Current division

Example 3-2

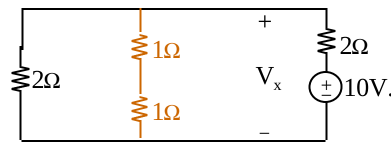
Find V_x



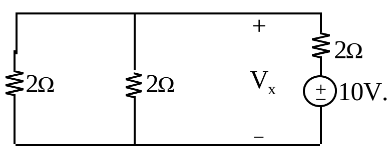
N.B.: See text Problem 3.9, p. 97.

Example 3-2 cont.

Step 1: 1Ω + 1Ω = 2Ω

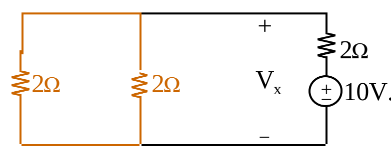


Example 3-2 cont.

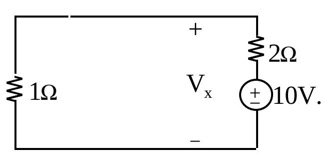


Example 3-2 cont.

Step 2: 2Ω | 2Ω = 1Ω



Example 3-2 cont.

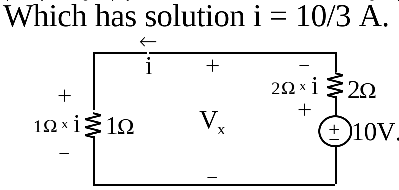


Example 3-2 cont.

Step 3: Single-loop circuit

KVL: 10 V. – 2Ω x i – 1Ω x i = 0 V.

Which has solution i = 10/3 A.



So what's V_x ?

3.3 Superposition

- *Superposition* is the defining (inherent) property of linear systems
- Ergo, *linearity* and *superposition* are synonymous
- Electric circuits are electrical systems
- The behavior of all voltages and currents in electric circuits consisting of constant valued resistors and (independent) voltage and/or current sources is governed by *linear algebraic equations* in these voltages and currents so superposition applies

3.3 Superposition cont.

What is the definition of the *Superposition* property?

If effect E_1 results from cause C_1
And if effect E_2 results from cause C_2
And if effect $K{\cdot}E_2$ results from cause $K{\cdot}C_2$
(where K is a constant)
Then cause $(C_1 + K{\cdot}C_2)$ results in effect $(E_1 + K{\cdot}E_2)$

And the cause-effect relationship (system) possessing the above *superposition* property is said to be *linear*

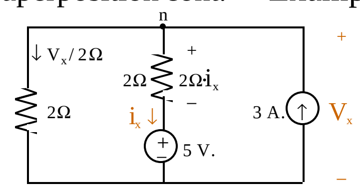
3.3 Superposition cont.

What's the practical use of this property; i.e., what's its application?

One-source-at-a-time superposition!

The key to the method's success lies in partitioning a given problem into single-source sub-problems (which are relatively easy to solve) and then applying superposition to the sub-problems' solutions—by just adding them up!

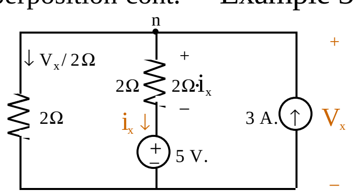
3.3 Superposition cont. —Example 3-3



N.B.: See text Problem 3.23, p. 99.

Find V_x and i_x using one-source-at-a-time superposition

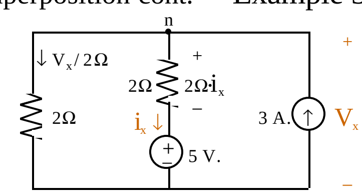
3.3 Superposition cont. —Example 3-3 cont.



N.B.: See text Problem 3.23, p. 99.

Step 1: Calculate the components of V_x and i_x that are caused by the 5 V. voltage source *only*. This begs the question "how is the influence of the 3 A. current source to be withdrawn from consideration?"

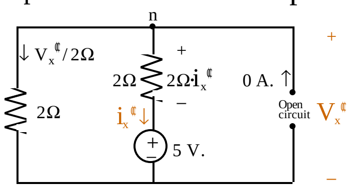
3.3 Superposition cont. —Example 3-3 cont.



N.B.: See text Problem 3.23, p. 99.

Step 1 (cont.): The answer is to replace it with a zero (no) Ampere current source (i.e., an open circuit)—no current, no influence!

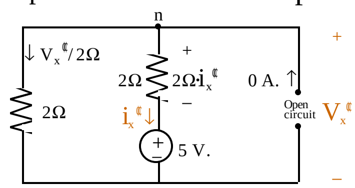
3.3 Superposition cont. —Example 3-3 cont.



N.B.: See text Problem 3.23, p. 99.

Step 1 (cont.): N.B.: The *partial* answers (caused by the 5 V. voltage source only) have been flagged with primes.

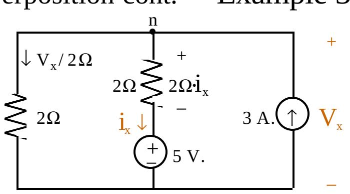
3.3 Superposition cont. —Example 3-3 cont.



N.B.: See text Problem 3.23, p. 99.

Step 1 (cont.): What's left is just a one-loop circuit having (by Ohm's Law) a counterclockwise current of $V_x' / 2\Omega = 5$ V. ÷ (2Ω + 2Ω) = 1.25 A. ∴ $V_x' = 2.5$ V. and KCL at node n yields $i_x' = -1.25$ A.

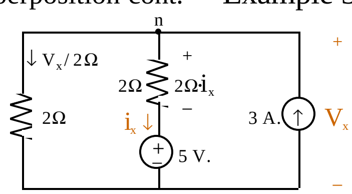
3.3 Superposition cont. —Example 3-3 cont.



N.B.: See text Problem 3.23, p. 99.

Step 2: Calculate the components of V_x and i_x that are caused by the 3 A. current source *only*. This begs the question "how is the influence of the 5 V. voltage source to be withdrawn from consideration?"

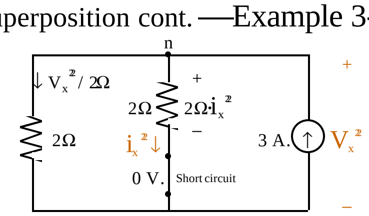
3.3 Superposition cont. —Example 3-3 cont.



N.B.: See text Problem 3.23, p. 99.

Step 2 (cont.): The answer is to replace it with a zero (no) Volt voltage source (i.e., a short circuit)—no voltage, no influence!

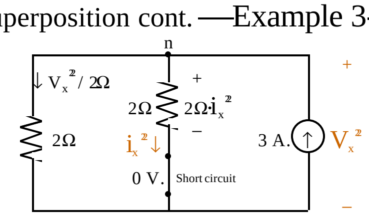
3.3 Superposition cont. —Example 3-3 cont.



N.B.: See text Problem 3.23, p. 99.

Step 2 (cont.): N.B.: The *partial* answers (caused by the 3 A. current source only) have been flagged with double primes.

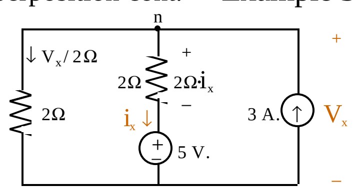
3.3 Superposition cont. —Example 3-3 cont.



N.B.: See text Problem 3.23, p. 99.

Step 2 (cont.): What's left is just a two-(equal) resistor current divider for which $i_x'' = [2\Omega / (2\Omega + 2\Omega)] \cdot 3$ A. = 1.5 A. so that $V_x'' = 2\Omega \cdot i_x'' = 2\Omega \cdot 1.5$ A. = 3 V.

3.3 Superposition cont. —Example 3-3 cont.



N.B.: See text Problem 3.23, p. 99.

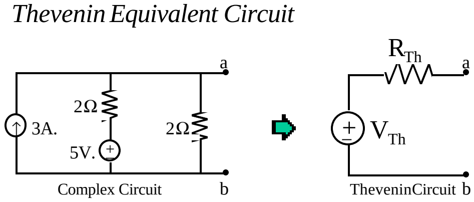
Step 3: Calculate each answer by summing its respective components; i.e., $i_x = i_x' + i_x'' = -1.25A. + 1.5A. = 0.25$ A.
and: $V_x = V_x' + V_x'' = 2.50V. + 3.0V. = 5.50$ V.

3.3 Superposition cont.

N.B.: Superposition *doesn't* work for power.

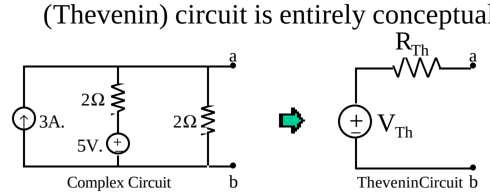
3.4 Thevenin and Norton Equivalent Circuits

The basic idea is to replace a relatively complex (linear) circuit with a simple *single-loop circuit* having a single voltage source and a single resistor—which is called the complex circuit's *Thevenin Equivalent Circuit*



3.4 Thevenin and Norton Equivalent Circuits

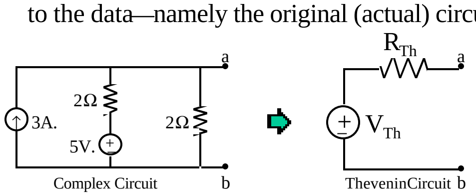
- What's needed are rules for how to calculate V_{Th} and R_{Th}
- Note that all of the numerical data is in the left (actual) circuit whereas the right (Thevenin) circuit is entirely conceptual



Data here *Concepts (theory) here*

3.4 Thevenin and Norton Equivalent Circuits

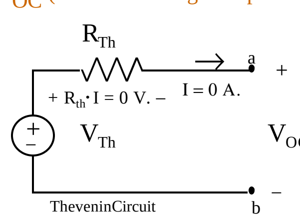
- This suggests that the rules for how to calculate V_{Th} and R_{Th} should be derivable from the Thevenin (theory) circuit and ...
- Once derived, these rules are then always applied to the data—namely the original (actual) circuit



Data here *Concepts (theory) here*

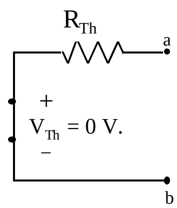
3.4 Thevenin Circuit Element Rules

KVL applied to the Thevenin circuit *with terminals a-b open* (so I = 0 A.) yields:
$V_{Th} - 0\ V. - V_{OC} = 0\ V.$ ∴
$V_{Th} = V_{OC}$ (Thevenin voltage = Open circuit voltage)



3.4 Thevenin Circuit Element Rules cont.

R_{Th} is the resistance of the circuit when all independent sources of energy are removed from consideration (all voltage sources shorted, all current sources opened)

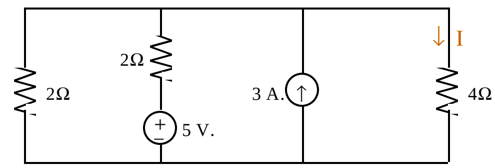


$R = R_{Th}$

3.4 Thevenin & Norton Equiv. Circuits cont.

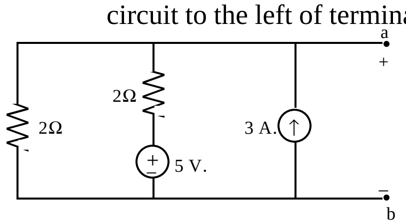
Example 3-4

Find I using Thevenin's Theorem



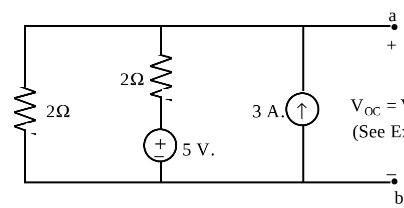
3.4 Example 3-4 cont.

Step 1: Get the Thevenin Equiv. of the circuit to the left of terminals a-b



3.4 Example 3-4 cont.

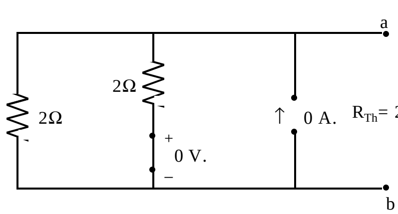
Step 1a: Open circuit voltage calculation



$V_{OC} = V_{Th} = 5.5$ V.
(See Example 3-3)

3.4 Example 3-4 cont.

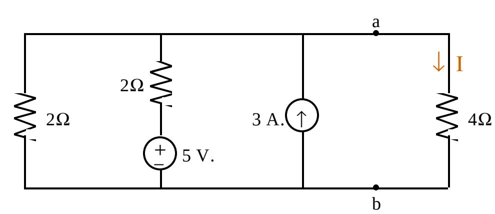
Step 1b: Determination of R_{Th}



$R_{Th} = 2\Omega \,|\, 2\Omega = 1\Omega$

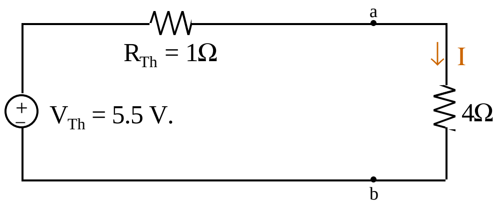
3.4 Example 3-4 cont.

Ergo, finding I using Thevenin's Theorem becomes



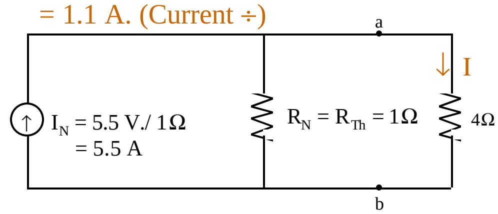
3.4 Example 3-4 cont.

$I = 5.5\text{ V.} / (1 + 4)\ \Omega = 1.1\text{ A.}$
(Ohm's Law)



3.4 Example 3-4 cont.

Using source conversions, the Norton solution is: $I = [1\Omega /(1 + 4)\Omega] \times 5.5\text{ A.} = 1.1\text{ A.}$ (Current ÷)

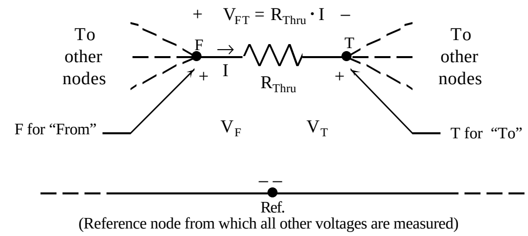


N.B.: See text pp. 76-81 for further information regarding Norton Equivalent circuits

3.5 Node & Mesh Analysis

- Node voltage analysis is just *organized* KCL(s)
- Likewise, mesh current analysis is just *organized* KVL(s)
- First consider node voltage analysis
- But before that, consider the following useful rule

From-to-through Rule

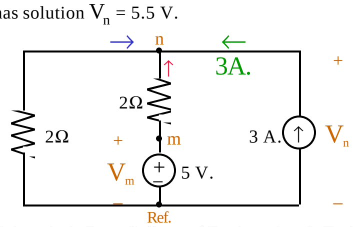


KVL: $V_F - V_{FT} - V_T = 0 \therefore V_{FT} = V_F - V_T = R_{Thru} \cdot I$ (Ω's Law)

$I = \dfrac{V_{From} - V_{To}}{R_{Thru}}$ ← Memorize this formula!

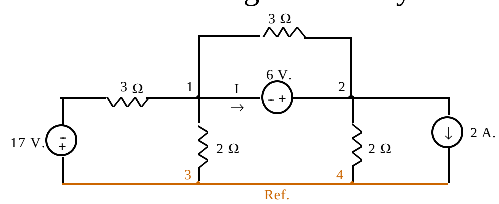
3.5 Node Voltage Analysis: Example 3-5

There are three nodes (m, n and the Ref. node)
V_m is known to be 5 V. (by inspection or by KVL)
Write KCLs at nodes where the voltage is *unknown*
KCL at node n: $(0\text{V.} - V_n)/2\Omega + (5\text{V.} - V_n)/2\Omega + 3\text{A.} = 0\text{ A.}$
—which has solution V_n = 5.5 V.



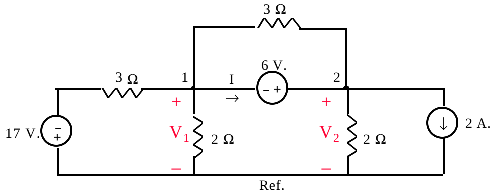
3.5 Node Voltage Analysis: Example 3-6

Find I using nodal analysis



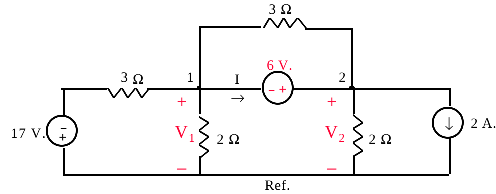
The first step is to (arbitrarily) establish the "bottom" (physical nodes 3 and 4) of the circuit as the (electrical) reference node (Ref.)

Example 3-6 cont.
(Find I using nodal analysis)



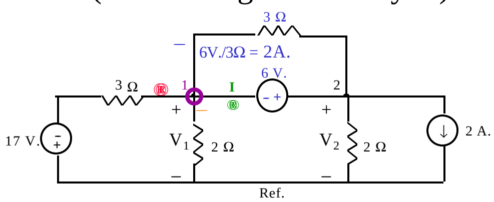
The node voltages V_1 and V_2 are with respect to the reference (bottom) node as shown

Example 3-6 cont.
(Find I using nodal analysis)



<u>KVL</u>: V_1 + 6 V. – V_2 = 0 V. ∴ $V_2 = V_1 + 6$ (A.)

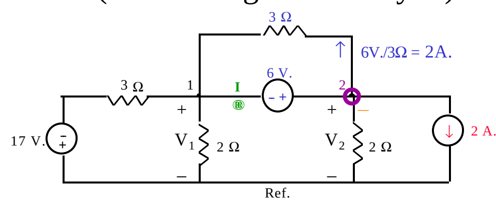
Example 3-6 cont.
(Find I using nodal analysis)



<u>KCL at Node 1</u>:

[(–17 V.) – V_1]/3Ω + 2 A. – I – V_1/2Ω = 0 A. (B.)

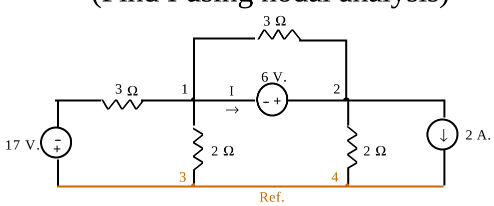
Example 3-6 cont.
(Find I using nodal analysis)



<u>KCL at Node 2</u>:

I – 2 A. – 2 A. – V_2/2Ω = 0 A. (C.)

Example 3-6 cont.
(Find I using nodal analysis)



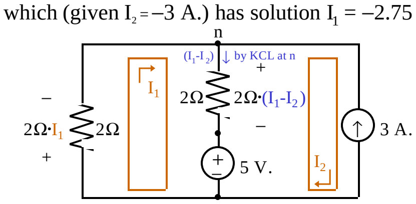
Equations (A.), (B.) and (C.) have solution:

V_1 = –8 V., V_2 = –2 V. and I = 3 A.

Ergo, I = 3 A.

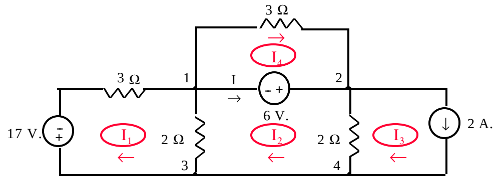
3.5 Mesh Current Analysis: Example 3-7

- There are two (left & right) meshes ("window panes")
- I_2 is known to be –3 A. (by inspection or by KCL)
- Write KVLs around meshes whose mesh currents are *unknown*:
- KVL (left mesh): $-2\Omega \cdot I_1 - 2\Omega \cdot (I_1 - I_2) - 5$ V. = 0 V. which (given I_2 = –3 A.) has solution I_1 = –2.75 A.



3.5 Mesh Current Analysis: Example 3-8

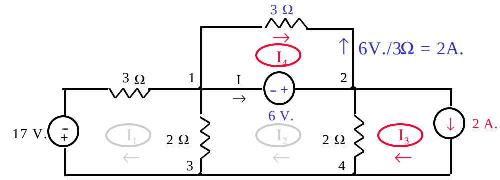
Find I using mesh analysis



The first step is to (arbitrarily) establish the mesh currents I_1, I_2, I_3 and I_4 as shown

Example 3-8 cont.

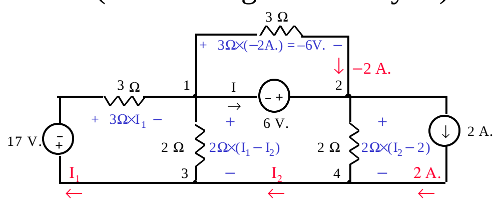
(Find I using mesh analysis)



Next, observe that I_3 = 2 A. and I_4 = –6V./3Ω = –2 A. and alter the schematic accordingly (see next slide)

Example 3-8 cont.

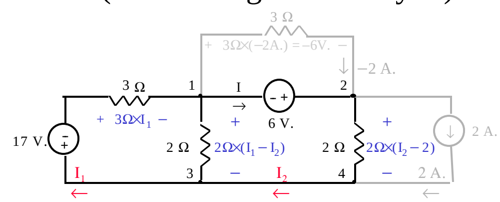
(Find I using mesh analysis)



Next, place *voltage drops* adjacent to the resistors as shown—note the use of the passive sign convention in assigning each voltage polarity!

Example 3-8 cont.

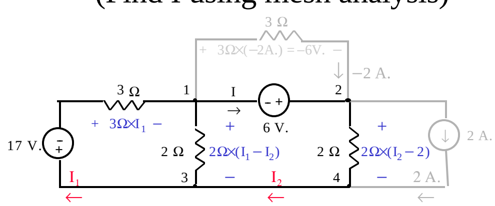
(Find I using mesh analysis)



Next, write KVLs around meshes whose mesh currents are *not* known

Example 3-8 cont.

(Find I using mesh analysis)



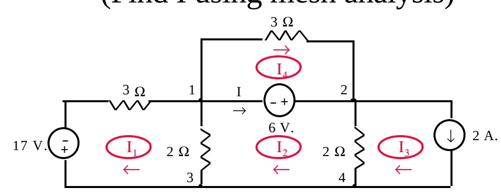
Mesh KVLs:

$$-17\text{ V.} - 3\Omega\times I_1 - 2\Omega\times(I_1 - I_2) = 0\text{ V.} \quad \text{(A.)}$$

$$2\Omega\times(I_1 - I_2) + 6\text{ V.} - 2\Omega\times(I_2 - 2) = 0\text{ V.} \quad \text{(B.)}$$

Example 3-8 cont.

(Find I using mesh analysis)



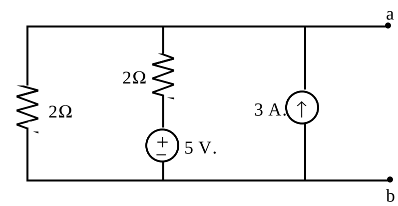
Equations (A.) and (B.) have solution I_1 = –3 A. and I_2 = 1 A. so given I_4 = –2 A., it follows that I = I_2 – I_4 = 1 A. – (–2 A.) = 3 A. ∴ I = 3 A.

3.6 Maximum Power Transfer

- To extract maximum power from a Thevenin circuit, load it with a resistance equal to R_{Th} — see text pp. 95-96 for (calculus) proof
- It follows then that to extract maximum power from *any* (linear) circuit, load it with its Thevenin resistance R_{Th} with respect to the extraction point

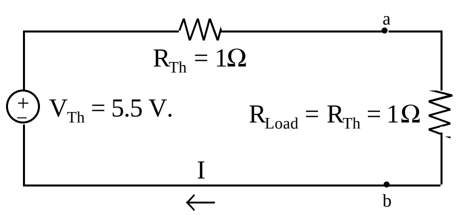
3.6 Maximum Power Transfer: Example 3-9

What's the maximum power that can be extracted from terminals a-b?



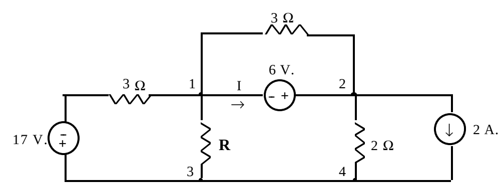
3.6 Maximum Power Transfer: Example 3-9 cont.

The circuit's Thevenin equivalent (found in Example 3-4) loaded with R_{Th} at terminals a-b yields:
$I = 5.5\ V./(1 + 1)\Omega = 2.75\ A.$ so the (maximum) load power is: $P_{max.} = I^2R = (2.75\ A.)^2 \times 1\ \Omega = 7.5625\ W.$



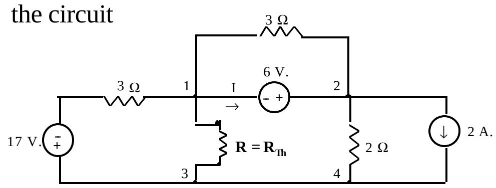
3.6 Maximum Power Transfer: Example 3-10

Determine the value of R in the circuit which will draw maximum power and calculate the corresponding maximum power.



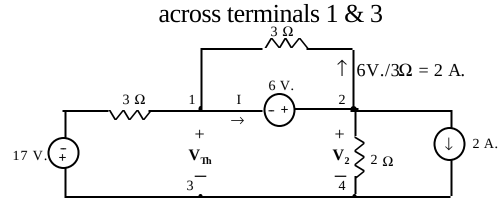
Example 3-10 cont.

<u>Solution strategy</u>: Determine the Thevenin circuit with respect to terminals 1 & 3 — then $R = R_{Th}$ will draw maximum power from the remainder of the circuit



Example 3-10 cont.

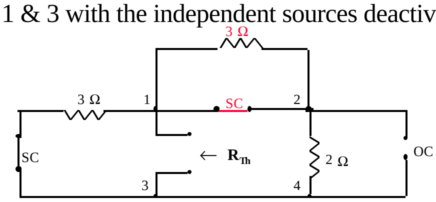
First find V_{Th} = open-circuit voltage across terminals 1 & 3



<u>KCL at Node 1</u>: $[(-17V.) - V_{Th}]/3\Omega + 2A. - I = 0\ A.$ (A.)
<u>KCL at Node 2</u>: $I - 2A. - V_2/2\Omega - 2A. = 0\ A.$ (B.)
<u>KVL</u>: $V_{Th} + 6V. - V_2 = 0\ V.$ (C.)
(A.), (B.) and (C.) have solution $V_{Th} = -12.8\ V.$

Example 3-10 cont.

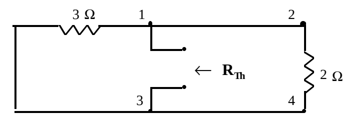
R_{Th} = Resistance across (open-circuited) terminals 1 & 3 with the independent sources deactivated



The parallel combination of the 0 Ω SC and the 3 Ω resistor is 0 Ω (another SC) so the circuit becomes (next slide) ...

Example 3-10 cont.

R_{Th} = Resistance across (open-circuited) terminals 1 & 3 with the independent sources deactivated



$R_{Th} = 3\,\Omega \parallel 2\,\Omega = 1.2\,\Omega$

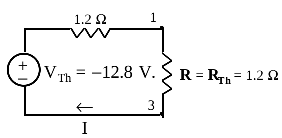
Example 3-10 cont.

The circuit's Thevenin equivalent loaded with $R = R_{Th}$ draws a current of:

$I = V_{Th}/(R_{Th} + R) = (-12.8\text{ V.})/(1.2\,\Omega + 1.2\,\Omega) = -5^1/_3\text{A.}$

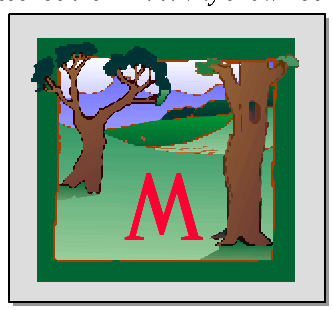
and the corresponding maximum power is

$P_{max.} = I^2R = (-5^1/_3\text{A.})^2\times 1.2\,\Omega = 34^2/_{15}\text{ W.} \approx 34.13\text{ W.}$

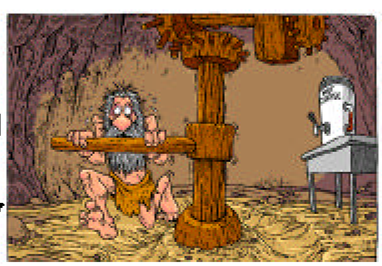


Examination

Describe the EE *activity* shown below



Homework



1O5 Problems ∴ ≈ 11 per week

Teamwork!

Exam Payoff

Acknowledgement:

Cartoons courtesy of

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We welcome your questions with

Enthusiasm!!