EE 313: Basic EE
Chapter 2 Synopsis

Chapter 2 Synopsis
Circuit Elements and Laws

Chapter 2: Introduction (pp. 9-10)

- Useful electromechanical devices have electric circuit counterparts (see next slide = text Figure 2.2)
- Ergo, electric circuit analysis is fundamental to designing and understanding the behavior of electromechanical and electronic devices and systems

Chapter 2: Introduction cont.

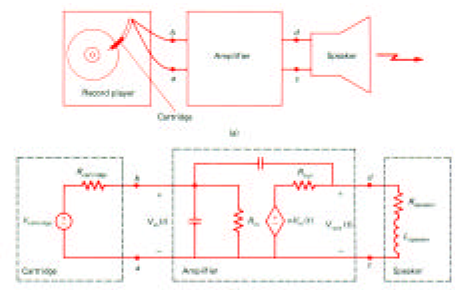


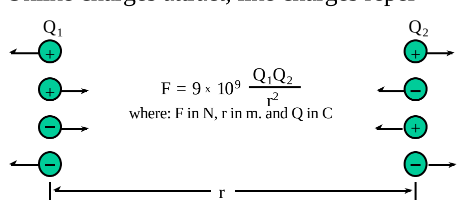
FIGURE 2.2 Modeling a physical electric circuit using ideal electric circuit elements: (a) actual circuit; (b) circuit ideal model

Chapter 2: Introduction (pp. 9-10)

- In electric circuit analysis the most fundamental quantities are voltages and currents
- These are interrelated and in many practical applications ideal (linear) relationships e.g., Ohm's Law ($V = R \times I$) apply

2.1 Charge and Electric Forces (pp. 11-13)

- Charge (Q_1 & Q_2) is the "stuff" of electricity
- There are two kinds of charge: positive (+) and negative (-)
- Unlike charges attract, like charges repel



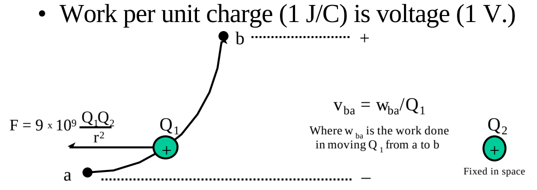
$$F = 9 \times 10^9 \frac{Q_1 Q_2}{r^2}$$

where: F in N, r in m. and Q in C

See Example 2.1 (pp. 12-13) for application

2.2 Voltage

- Charges exert *forces* on one another
- ∴ *Moving* a charge Q_1 in the presence of another charge Q_2 entails doing *work* (expending energy) on that charge (Q_1)
- Work per unit charge (1 J/C) is voltage (1 V.)

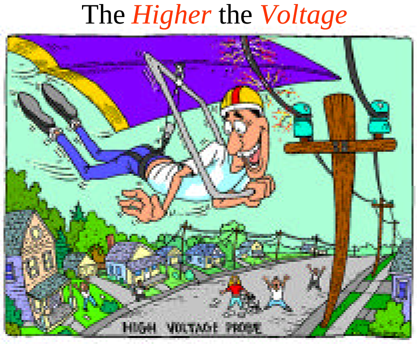


See Example 2.2 (pp. 15-16) for application

2.2 Voltage cont.

- Another interpretation of voltage is:
  Charge *carries* energy: 1 J/C is 1 V.
- How much *energy* does a 12-V. automobile battery impart to each Coulomb of *charge* passing through it?

The *More Energy* per-unit charge, The *Higher* the *Voltage*

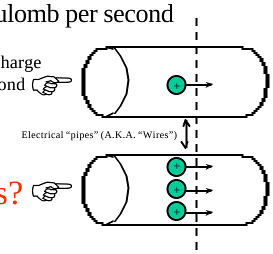


2.3 Current

- Current is a measure of the rate of flow of (net positive) charge per unit time
- 1 Ampere = 1 Coulomb per second

If this represents 1 C. of charge crossing the line each second
A.K.A. 1 Ampere . . .

What's this?



2.3 Current cont.

- Current is a measure of the rate of flow of (net positive) charge per unit time
- 1 Ampere = 1 Coulomb per second
- See next slide = Figure 2.10 (p. 17) for a graphical illustration of the charge/current relationship

$$i(t) = \frac{dq(t)}{dt} \qquad (2.4)$$

$$q(t) = \int_{-\infty}^{t} i(\tau)d\tau \qquad (2.5)$$

Units: i(t) is in Amperes when q(t) is in Coulombs and t is in seconds

2.3 Current cont.

$$q(t) = \int_{-\infty}^{t} i(\tau)d\tau$$

$$i(t) = \frac{dq(t)}{dt}$$

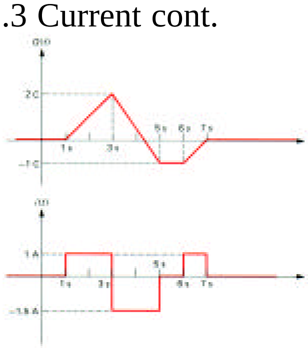
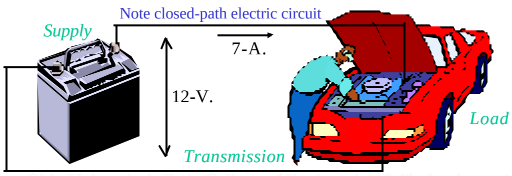


FIGURE 2.10 Relation between current and net positive charge flowing past a point.

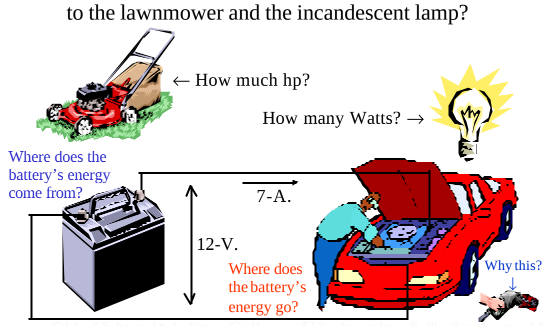
2.3 Voltage & Current Together

- Charge *carries* energy: 1 J/C is 1-V.
- Charge in *motion* is current: 1 C/Sec. is 1-A.
- How much *power* does the battery deliver to the automobile? (What's power?)
- How much *energy* does the battery deliver to the automobile? (Over what time interval?)



2.3 Voltage & Current Together cont.

How do the battery's power and energy compare to the lawnmower and the incandescent lamp?



2.3 Recapitulation: V & I Together

- Charge *carries* energy: 1 J/C is 1-V.
- Charge in *motion* is current: 1 C/Sec. is 1-A.

1. Where's the closed circuit?*
2. Why is Mr. DVM upset?**

* Ergo, what's wrong with the cartoon's physics?

** Give a *quantitative* answer in terms of the lawnmower & light bulb! (Hint: What's the voltage and what's a reasonable current?)

2.3 Recapitulation: V & I Together

- Which scenario is more dangerous, HV or household?

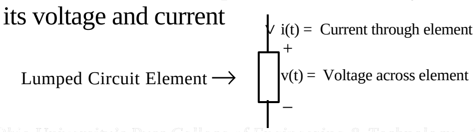
(N.B.: The answer isn't as clear-cut as one might think.)

e.g., ~ 8-KV

$120\text{-}V_{rms}$

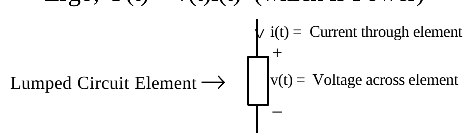
2.4 Lumped-Circuit Elements

- Macroscopic (big picture) treatment of electrophysics
- Physical behavior of a region of electrical activity is averaged (lumped together) into a so-called "lumped-circuit element"
- The overall electrical activity associated with the element is captured, inter alia, by its voltage and current



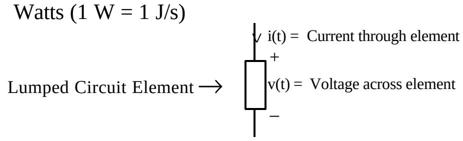
Voltage-Current -Power Relationship

- Voltage units are Joules / Coulomb
- Current units are Coulombs / second
- Voltage-Current product's units are …
- (Joules / Coulomb) x (Coulombs / second) …
- = Joules / second = Watts
- Ergo, P(t) = v(t)i(t) (which is Power)



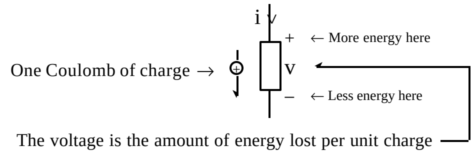
Passive Sign Convention

- Given the passive sign convention shown below ...
- Then *if* at some time "t" *both* i(t) > 0 and v(t) > 0
- *Then* positive charge is moving through the element from top-to-bottom at the rate i(t) (in C/sec.) *and* …
- The rate (per unit charge) of work (energy) being done to push the charge through the element is v(t) (in J/C)
- The power *absorbed* by the element is p(t) = v(t)i(t) in Watts (1 W = 1 J/s)

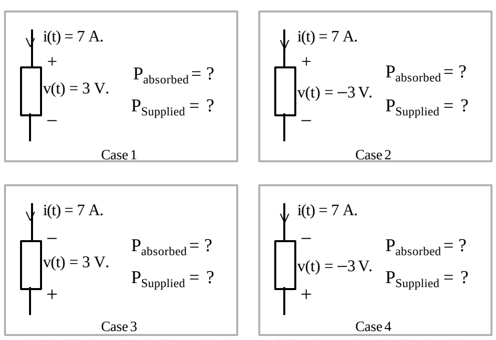


Voltage Polarity Meaning

- The amount of energy (in Joules) that *each* Coulomb of charge looses (expends), as a result of its journey from top-to-bottom through the element, is numerically equal to its voltage (in Volts)



Power Supplied vs. Power Absorbed

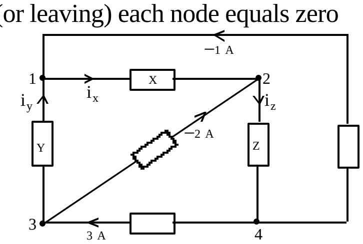


2.5 Kirchhoff's Voltage and Current Laws

- The behavior of electric-circuit currents and voltages are governed by these laws
- KCL is a manifestation of conservation of charge (see text p. 22)
- KVL is a manifestation of conservation of energy (see text p. 25)
- KCL and KVL are used in conjunction with circuit element laws; e.g., Ohm's Law (see text p. 28), etc. to perform electric circuit analysis

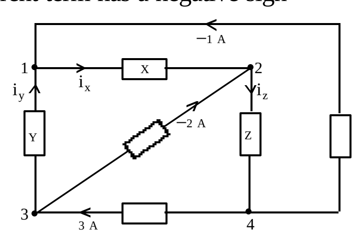
Kirchhoff's Current Law (KCL)

- A circuit node is an electrical "location" of interest (see numbered black dots below)
- KCL: The sum-total of all currents entering (or leaving) each node equals zero



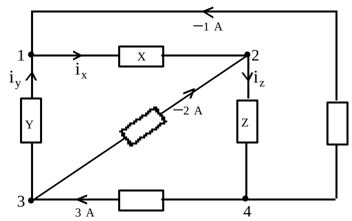
Kirchhoff's Current Law (KCL) cont.

- KCL Sum: Each term in the sum is a current
- If summing *into* a node, each inbound current term has a positive sign and each outbound current term has a negative sign



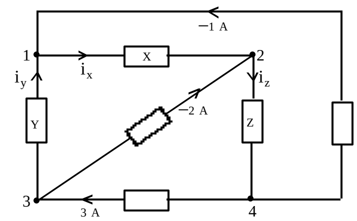
Kirchhoff's Current Law (KCL) cont.

- KCL at Node 1: $+ i_y - i_x + (-1\text{ A}) = 0\text{ A}$ (I)
- KCL at Node 2: $+ i_x + (-2\text{ A}) - i_z = 0\text{ A}$ (II)
- KCL at Node 3: $- i_y - (-2\text{ A}) + 3\text{ A} = 0\text{ A}$ (III)



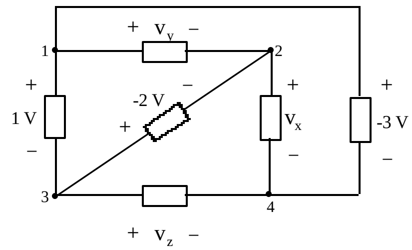
Kirchhoff's Current Law (KCL) cont.

- (I), (II) & (III) have solution $i_x = 4$ A, $i_y = 5$ A and $i_z = 2$ A.
- Note that doing a KCL at node 4 is now redundant



Kirchhoff's Voltage Law (KVL)

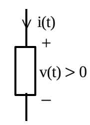
- Relates a circuit's voltages to one another
- First must consider voltage *rise* vs. voltage *drop* concept



Voltage Rise vs. Voltage Drop

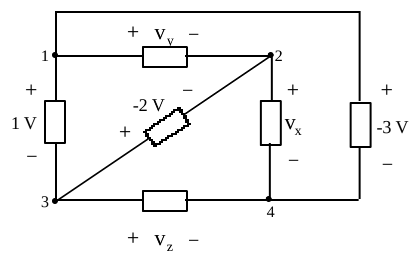
- Positive charge moving across a change of voltage from the positive polarity sign towards the negative polarity sign experiences a voltage *drop* (a decrease in its energy)
- Positive charge moving across a change of voltage from the negative polarity sign towards the positive sign experiences a voltage *rise* (an increase in its energy)

If $i(t) > 0$, the downward moving charge experiences a voltage *drop*. However, if $i(t) < 0$ then the upward moving charge experiences a voltage *rise*.



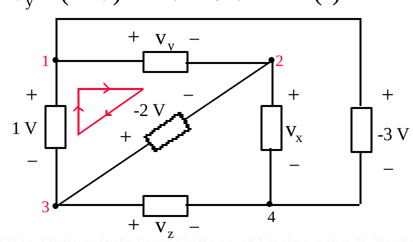
Kirchhoff's Voltage Law (KVL)

- KVL: The sum-total of all voltage drops and voltages rises around any *closed* path—in any direction—equals zero



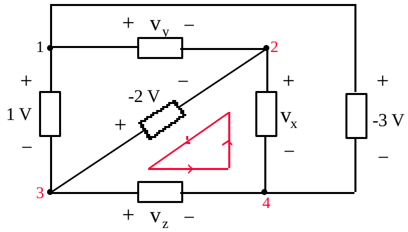
Kirchhoff's Voltage Law (KVL)

- KVL for closed path 1→2→3→1 (using the "land on polarity sign" rule to determine the sign of each term in the sum):
- $- v_y + (-2\text{ V}) + 1\text{ V} = 0\text{ V}$ (I)



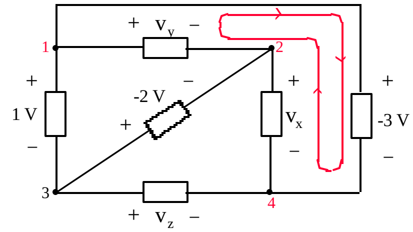
Kirchhoff's Voltage Law (KVL)

- KVL for closed path $2 \rightarrow 3 \rightarrow 4 \rightarrow 2$ (using "land on polarity sign" rule to determine the sign of each term in the sum):
- $+ (-2\ V) - v_z + v_x = 0\ V$ (II)



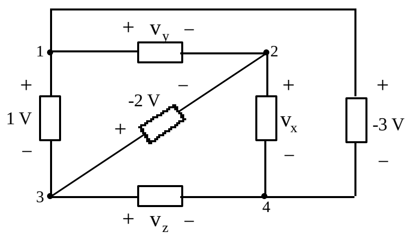
Kirchhoff's Voltage Law (KVL)

- KVL for closed path $1 \rightarrow 4 \rightarrow 2 \rightarrow 1$ (using "land on polarity sign" rule to determine the sign of each term in the sum):
- $- (-3\ V) + v_x + v_y = 0\ V$ (III)

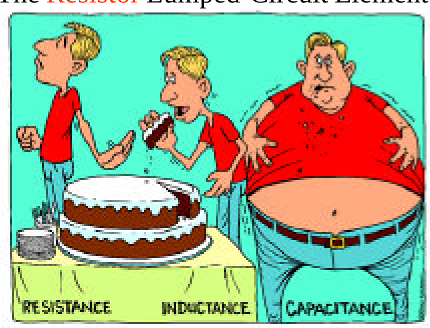


Kirchhoff's Voltage Law (KVL)

- (I), (II) and (III) can be solved for v_x, v_y and v_z
- N.B.: There are other (now redundant) closed paths (also see Text Problem 2.23, p. 52)



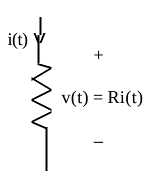
The Resistor Lumped-Circuit Element



R ***L*** ***C***

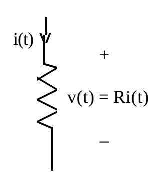
2.6 The Resistor

- The (ideal) resistor is defined by Ohm's Law: $v(t) = Ri(t)$ where R is a positive constant *and*
- $v(t)$ and $i(t)$ are *also* related by the passive sign convention (this second requirement is implicitly built into the Ohm's Law equation)



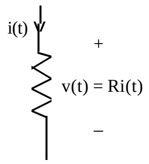
2.6 The Resistor cont.

- If $i(t) > 0$ so too is $v(t) = Ri(t)$ and vice versa; i.e., if $i(t) < 0$ so is $v(t)$.
- So the resistor's absorbed power (voltage-current product) is always $P_{absorbed} = v(t)i(t) \geq 0$



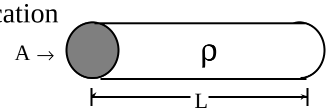
2.6 The Resistor cont.

- $G = R^{-1}$ is called conductance (in S)
- $P_R(t) = vi = (Ri)i = i^2R$
- $P_R(t) = vi = v(v/R) = v^2/R$



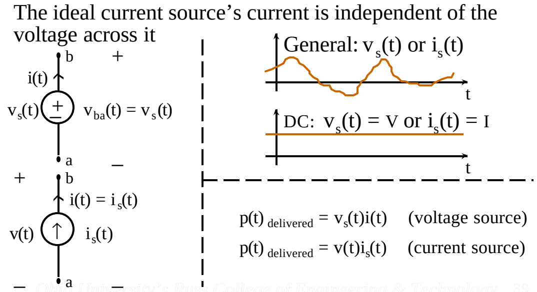
2.6 The Resistor cont. (more about R)

- $R = \rho(L/A)$ where
- The resistivity (ρ) is material dependent (in $\Omega \cdot m$)
- The length (L) is in m.
- The cross-sectional area (A) is in m^2.
- See Text Example 2.4 (p. 30) for an application



2.7 Voltage and Current Sources

- In circuit analysis, the ideal source concept is fundamental
- The ideal voltage source's terminal voltage is independent of the source's current
- The ideal current source's current is independent of the voltage across it

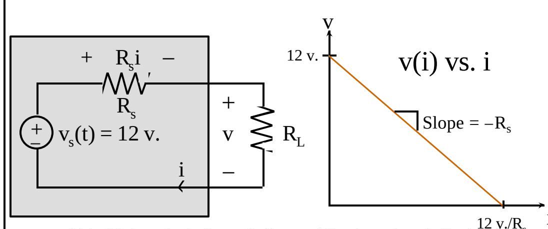


$p(t)_{delivered} = v_s(t)i(t)$ (voltage source)

$p(t)_{delivered} = v(t)i_s(t)$ (current source)

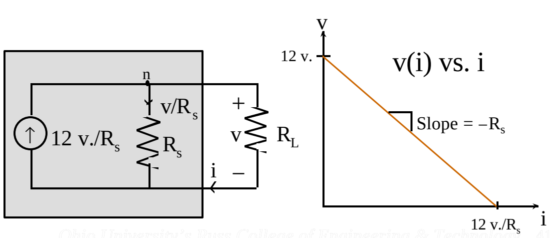
2.7 Source modeled via ideal source & resistor

- An actual source can be modeled using ideal circuit elements (e.g., ideal source & resistor—see below)



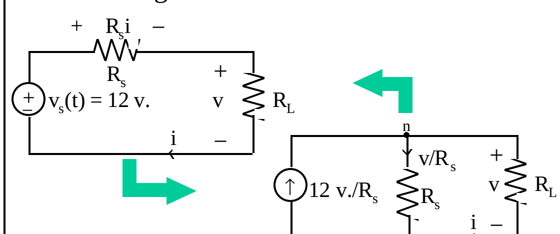
2.7 Source modeled via ideal source & resistor

- An actual source can be modeled using ideal circuit elements (e.g., ideal source & resistor—see below)

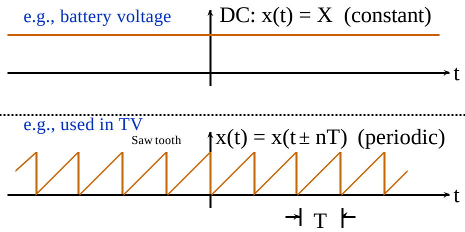


2.7 Source modeled via ideal source & resistor

- The preceding *source transformations* should be committed to memory, according to the text!



2.8 Signal waveforms



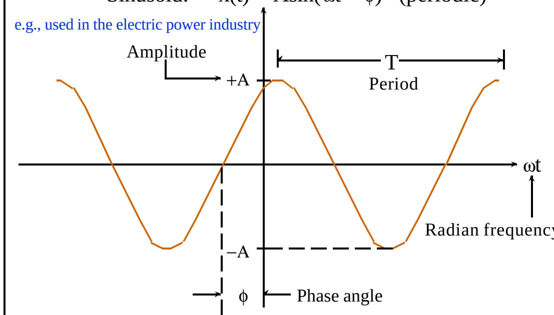
n = 1, 2, 3, … (integer)

T = Period (in sec.)

f = 1/T Frequency (in Hz.)

2.8 Signal waveforms cont.

Sinusoid: $x(t) = A\sin(\omega t + \phi)$ (periodic)



2.8 Signal waveforms cont.

- Waveforms can be *numerically* compared (as opposed to *visually* compared) by way of numerical attributes which measure some meaningful waveform property
- Average value is intuitively appealing (it measures a waveform's DC content)

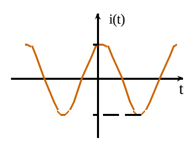
$$x_{AVE} = \frac{1}{T}\int_{t}^{t+T} x(\tau)d\tau$$

(Where T is x(t)'s period)

2.8 Signal waveforms cont.

If the AC's average is zero,

why's Mr. DVM upset?*



* Moral: Average current isn't of much interest here (and is perhaps misleading).

2.8 Signal waveforms cont.

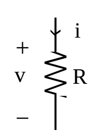
- The RMS (Root-Mean-Square) value measures a *periodic* waveform's *average* power
- RMS values of sinusoidal voltages and currents are the vernacular of the electric power industry

$$x_{RMS} = \sqrt{\frac{1}{T}\int_{t}^{t+T} x^2(\tau)d\tau}$$

(Where T is x(t)'s period)

2.8 Signal waveforms cont.

- A *current* signal's RMS value is a measure of its *average* power



$$P_{AVE} = \frac{1}{T}\int_{t}^{t+T} p(\tau)d\tau$$

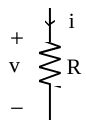
$$P_{AVE} = \frac{1}{T}\int_{t}^{t+T} Ri^2(\tau)d\tau$$

$$P_{AVE} = R\left[\frac{1}{T}\int_{t}^{t+T} i^2(\tau)d\tau\right] = I^2_{RMS}$$

$$P_{AVE} = RI^2_{RMS} \quad \text{(QED)}$$

2.8 Signal waveforms cont.

- A *voltage* signal's RMS value is a measure of its *average* power



$$P_{AVE} = \frac{1}{T}\int_t^{t+T} p(\tau)d\tau$$

$$P_{AVE} = \frac{1}{T}\int_t^{t+T} v^2(\tau)/R\,d\tau$$

$$P_{AVE} = \frac{1}{R}\left[\frac{1}{T}\int_t^{t+T} v^2(\tau)d\tau\right] \quad (= V^2_{RMS})$$

$$P_{AVE} = V^2_{RMS} / R \quad \text{(QED)}$$

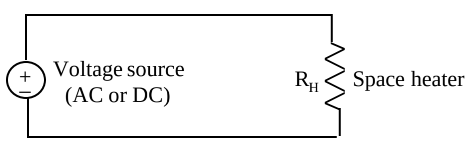
2.8 Signal waveforms cont.

- Moral: A periodic signal's RMS value measures how powerful the signal is
- See text p. 38 for a sample RMS value calculation
- Another important RMS result (see text p. 39) is that the RMS value of the general sinusoidal waveform $x(t) = A\sin(\omega t + \phi)$ is $A/\sqrt{2} \approx 0.707A$
- Ergo, it is easy to program meters (e.g., VOMs, DVMs, etc.) to display the RMS values of the AC (sinusoidal) waveforms encountered in the electric power industry

2.8 Signal waveforms cont.

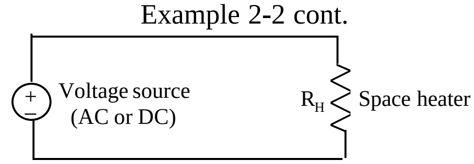
Example 2-2

The resistance space heater (of constant resistance R_H) when supplied by a $170\sin(120\pi t)$ V. (t in seconds) sinusoidal (AC) voltage source takes 5 minutes to raise a room's temperature 3° F. If the same heater is supplied by a 48 Volt (DC) voltage source, how long will it take (in minutes) to do the same job?



2.8 Signal waveforms cont.

Example 2-2 cont.



E_{AC} = Energy for AC case = $P_{ave.} \times T_{AC} = [(V_{RMS})^2/R_H] \times 5$ Min.

$\therefore E_{AC} = [(170\text{ V.}/\sqrt{2})^2/R_H] \times 5$ Min.

E_{DC} = Energy for DC case = $P_{DC} \times T_{DC} = [(V_{DC})^2/R_H] \times T_{DC}$

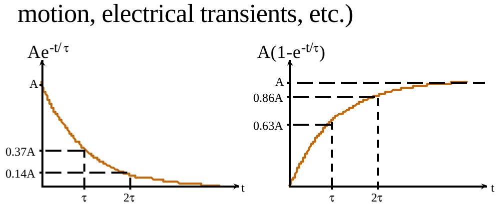
$\therefore E_{DC} = [(48\text{ V.})^2/R_H] \times T_{DC}$

Then: $E_{AC} = [(170\text{ V.}/\sqrt{2})^2/R_H] \times 5\text{ Min.} = [(48\text{ V.})^2/R_H] \times T_{DC} = E_{DC}$

which yields $T_{DC} \approx$ 31 Min.

2.8 Signal waveforms cont.

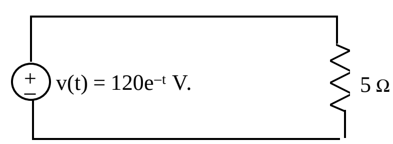
- Exponential waveform: $x(t) = Ae^{-t/\tau}$ (t is in seconds)
- Time constant (parameter) is τ (also in seconds)
- Captures *transient* behavior of many physical phenomena (e.g., radioactive decay, mechanical motion, electrical transients, etc.)



2.8 Signal waveforms cont.

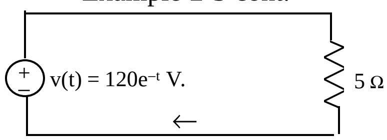
Example 2-3

An ideal exponential voltage source of $120e^{-t}$ V. supplies power to a 5 Ω resistor. Calculate the following quantities for the time period $0 \le t < \infty$: the total energy (in Joules) absorbed by the resistor, the total charge (in Coulombs) that flows through the resistor, the average voltage (in Volts) across the resistor and the time it takes an AC, 120 V. (RMS), 60 W., incandescent lamp to consume the same energy as the resistor.



2.8 Signal waveforms cont.

Example 2-3 cont.



$i(t) = v(t)/5\Omega = 24e^{-t}$ A.

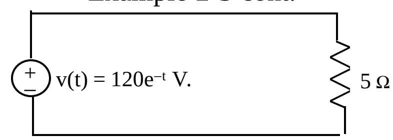
Power: $p(t) = v(t) \times i(t) = 5\Omega \times i^2(t) = v^2(t)/5\Omega = 2880e^{-2t}$ W.

Energy: $W_R = \int_0^{\infty} p(\tau)d\tau = \int_0^{\infty} 2880e^{-2\tau}\,d\tau = 1.44$ kJ.

Charge: $Q = \int_0^{\infty} i(\tau)d\tau = \int_0^{\infty} 24e^{-\tau}\,d\tau = 24$ C.

2.8 Signal waveforms cont.

Example 2-3 cont.



The average voltage is: $V_{ave.} = W/Q = 1.44$ kJ./24C. = 60 J/C

$\therefore V_{ave.} = 60$ V. (in what sense is this the voltage's *average*?)

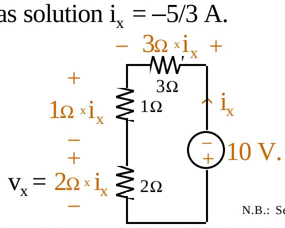
Equating the lamp's energy to the resistor's energy yields:

$P_{lamp} \times T_{lamp} = 1.44$ kJ. $= W_{5\Omega}$ (where $P_{lamp} = 60$ W. = 60 J./Sec.)

Ergo, $T_{lamp} = 24$ Seconds

2.9 Analysis of Simple Circuits

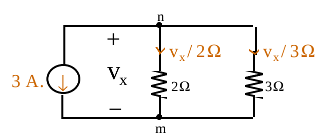
- *Single-loop circuit analysis* involves several applications of Ohm's Law ($1\Omega \times i_x$, $2\Omega \times i_x$ and $3\Omega \times i_x$ (note use of passive sign convention on schematic)) and ...
- One KVL: $2\Omega \times i_x + 1\Omega \times i_x + 3\Omega \times i_x + 10$ V. = 0 V. which has solution $i_x = -5/3$ A.



N.B.: See text Problem 2.31, p. 54.

2.9 Analysis of Simple Circuits cont.

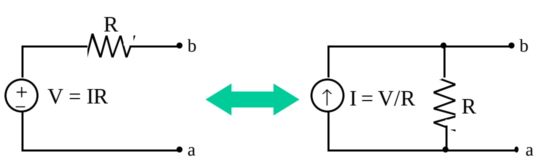
- *Single-node-pair circuit analysis* involves several applications of Ohm's Law ($v_x / 2\Omega$ and $v_x / 3\Omega$)—note use of passive sign convention on schematic and ...
- One KCL (at node n or m): $3\text{ A.} + v_x / 2\Omega + v_x / 3\Omega = 0$ A. which has solution $v_x = -3.6$ V.



N.B.: See text Problem 2.30, p. 53.

2.9 Analysis of Simple Circuits cont.

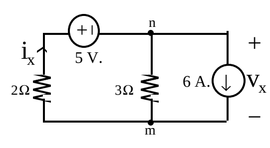
- *Source transformations* (described earlier and repeated below) can be used to convert some circuits to either a *single-loop circuit* or *single-node-pair circuit*—which can then be analyzed as described earlier



2.9 Analysis of Simple Circuits cont.

Example 2-4

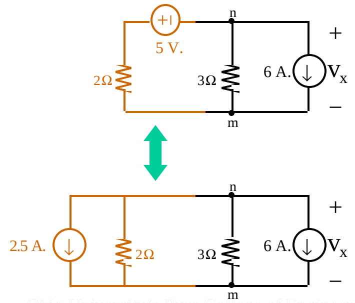
Find i_x and v_x



N.B.: See text Problem 2.39, p. 56.

Example 2-4 cont.

- Step 1: Create single-node-pair circuit using source conversion

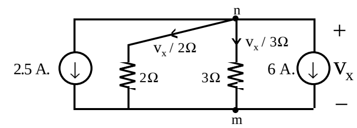


Example 2-4 cont.

- Step 2: Solve the single-node-pair circuit for v_x
- KCL at node n:

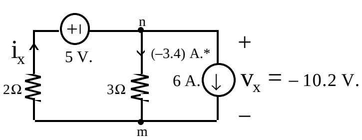
$$2.5\text{ A.} + v_x / 2\Omega + v_x / 3\Omega + 6\text{ A.} = 0\text{ A.}$$

- Which has solution $v_x = -10.2$ V.



Example 2-4 cont.

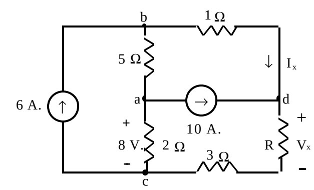
- Step 3: With v_x now known, solve the original circuit for i_x
- KCL at node n: $i_x - (-3.4)\text{ A.} - 6\text{ A.} = 0\text{ A.}$
- Which has solution $i_x = 2.6$ A.



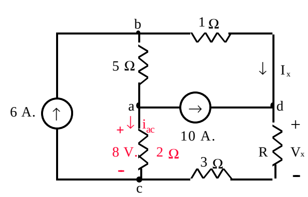
*N.B.: (–10.2 V. / 3Ω) = –3.4 A.

2.9 Analysis of Simple Circuits cont.

Example 2-5: Find I_x and V_x

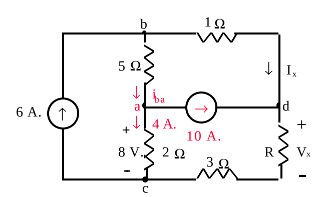


Example 2-5 cont.



<u>Ohm's Law</u>: $i_{ac} = 8\text{ V.} / 2\,\Omega = 4\text{ A.}$

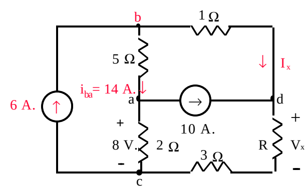
Example 2-5 cont.



<u>KCL at node a</u>: $i_{ba} - 4\text{ A.} - 10\text{ A.} = 0\text{ A.}$

$\therefore\ i_{ba} = 14\text{ A.}$

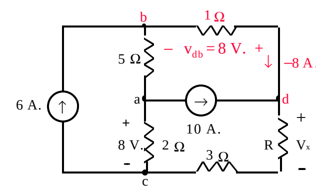
Example 2-5 cont.



KCL at node b: 6 A. – 14 A. – I_X = 0 A.

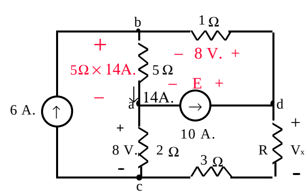
∴ $I_X = -8$ A.

Example 2-5 cont.



Ohm's Law: v_{db} = 1 Ω × [−(− 8 A.)] = 8 V.

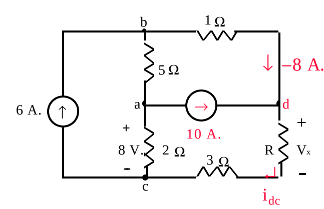
Example 2-5 cont.



KVL (Top-right mesh): 5Ω × 14A. + 8 V. – E = 0 V.

∴ E = 78 V.

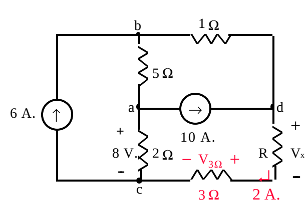
Example 2-5 cont.



KCL at node d: 10 A. + (–8 A.) – i_{dc} = 0 A.

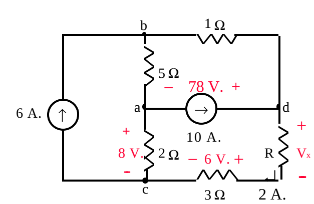
∴ i_{dc} = 2 A.

Example 2-5 cont.



Ohm's Law: $V_{3\Omega}$ = 3 Ω × 2 A. = 6 V.

Example 2-5 cont.



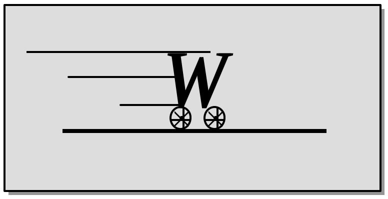
KVL (Bottom-right mesh):

8 V. + 78 V. – V_X – 6 V. = 0 V.

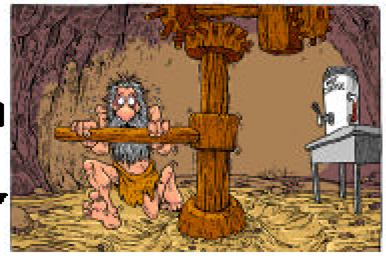
∴ V_X = 80 V.

Examination

Identify the *dynamic* EE object shown below



H o m e w o r k



1O5 Problems ∴ ≈ 11 per week

Teamwork!

E x a m P a y o f f

Acknowledgement:

Cartoons courtesy of

We welcome your questions with Enthusiasm!!