

**Microsoft PowerPoint<sup>®</sup> Presentation Graphics for  
EE 315: Basic Electrical Engineering III**

Prepared by Brian Manhire, Ph.D.  
Professor of Electrical Engineering



*Stocker Center, home of Ohio University's  
Russ College of Engineering & Technology*

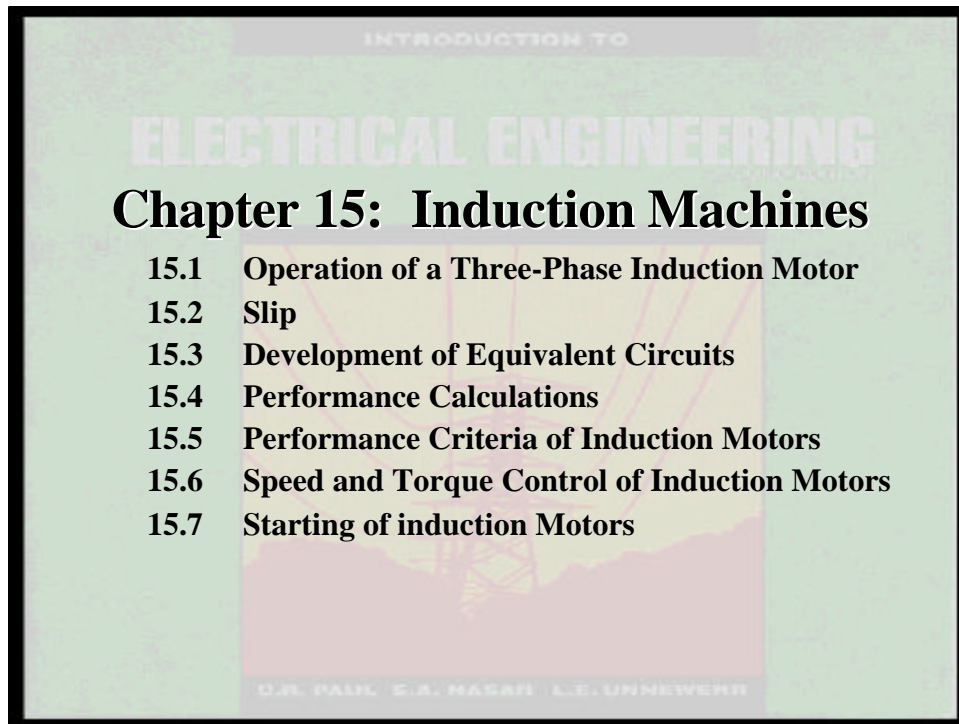
Microsoft PowerPoint<sup>®</sup> Presentation Graphics  
© Copyright 1998 Brian Manhire

For Part 3 of

**Introduction to Electrical  
Engineering, 2/e**

by **C.R. Paul, S.A. Nasar  
and L.E. Unnewehr**

© **1992**, McGraw-Hill, Inc.



### Section 15.1: Operation of a Three-Phase Induction Motor

Induction motors are, by far, the most widely used motors

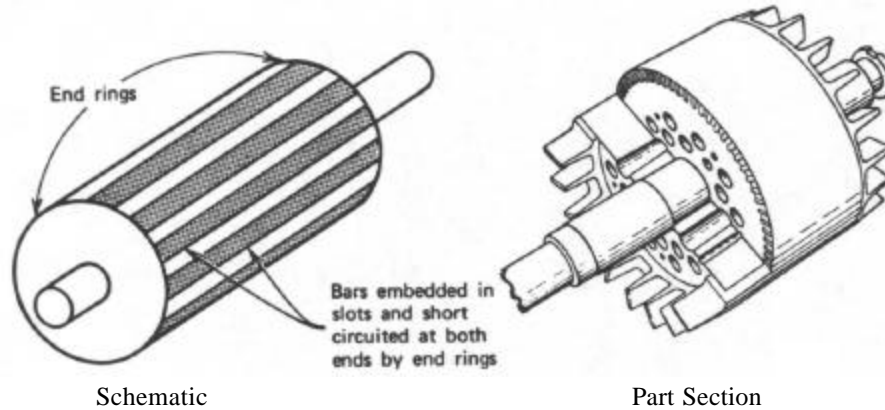
AC windings are mounted on the (rotating) rotor

AC windings are mounted on the (stationary) stator

resulting in three-phase AC stator voltages and currents—which, like a three-phase synchronous machine, produces a rotating field

There are two basic rotor designs, namely, (squirrel) cage (extremely rugged) which is the most common design, and wound-rotor, which is a more versatile but more expensive and fragile design—which are illustrated in the next two slides

Section 15.1: Operation of a Three-Phase Induction Motor cont.

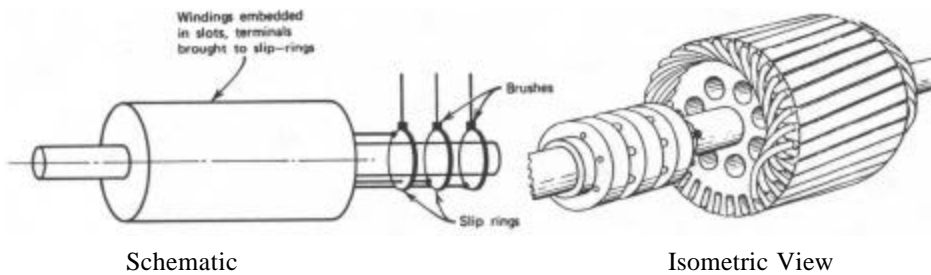


Source: S. Nasar, *Electric Machines and Power Systems: Volume I, Electric Machines*, McGraw-Hill, New York, 1995, p. 144.

**Squirrel-Cage Rotor**

N.B.: Rotor is not electrically accessible

Section 15.1: Operation of a Three-Phase Induction Motor cont.



Source: S. Nasar, *Electric Machines and Power Systems: Volume I, Electric Machines*, McGraw-Hill, New York, 1995, p. 144.

**Wound Rotor**

N.B.: Rotor electrically accessible via slip rings

Section 15.1: Operation of a Three-Phase Induction Motor cont.

The stator's rotating field *cuts* the rotors conductors thereby inducing three-phase voltages in the rotor circuit

The three-phase induced (Faraday) voltages cause three-phase currents to flow in the rotor

The rotor's three-phase currents produce a rotating (rotor) field which is always aligned (travels with) the stator's rotating field

The whole process is essentially that of a transformer

Ergo, the induction motor is sometimes referred to, in the vernacular, as a *rotating transformer*

The rotor structure chases the rotating stator field—but can never catch up to it because of electrical friction (rotor resistance)

12:23 PM Ohio University's Russ College of Engineering & Technology 7

## Section 15.2: Slip

Given the rotor's speed  $n$ , and the stator's rotating field (synchronous) speed  $n_s = 120f/P$  (see Eq. 14.3, text p. 536), the slip is:  $s = (n_s - n)/n_s$

The numerator ( $n_s - n$ ) is how much faster the stator field is rotating than the rotor (relative motion) and  $n = (1 - s)n_s$

The relative motion between the stator field and the rotor determines how frequently the rotating stator field cuts the rotating rotor conductors—so the frequency of the rotor currents is:  $f_R = sf$

The frequency of the rotor currents determines the speed of the rotor field with respect to the rotor:  $n_{Rf} = 120 f_R / P = 120 (sf) / P = sn_s$

The speed of the rotor field with respect to the *stationary* stator structure is the rotor speed plus the rotor field's speed with respect to the rotor which is:  $n + n_{Rf} = (1 - s)n_s + sn_s = n_s$

Ergo, the rotor field and the stator field rotate together at the synchronous speed

12:23 PM Ohio University's Russ College of Engineering & Technology 8

## Section 15.2: Slip Example 15.1

A six-pole three-phase 60 Hz induction motor runs at 4% slip at a certain load. Calculate: the synchronous speed, the rotor speed, the frequency of the rotor currents, the speed of the rotor field with respect to the stator and speed of the rotor field with respect to the stator field.

The synchronous speed is:  $n_s = 120f/p = 120 \times 60/6 = 1200 \text{ RPM}$

The rotor speed is  $n = (1 - s)n_s = (1 - 0.04) \times 1200 = 1152 \text{ RPM}$

$f_R = sf = 0.04 \times 60 = 2.4 \text{ Hz.}$

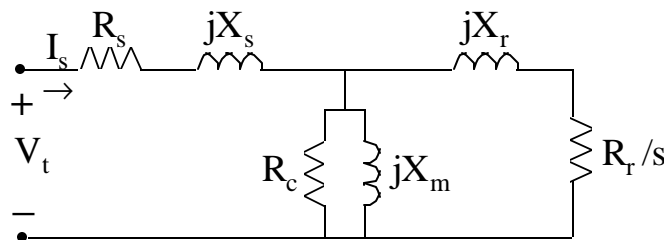
The speed of the rotor field with respect to the (rotating) rotor structure is  $n_{Rf} = 120 f_R / P = 120 (sf) / P = sn_s = 0.04 \times 1200 = 48 \text{ RPM}$

The speed of the rotor field with respect to the (stationary) stator structure is  $n + n_{Rf} = (1 - s)n_s + sn_s = n_s = 1200 \text{ RPM}$   
(i.e., the rotor and stator fields rotate together)

## Section 15.3: Development of Equivalent Circuits

Since the induction motor is a *rotating transformer*, its (per-phase A) 60 Hz. phasor-equivalent-circuit is (similar to that of a transformer) as shown below (see text pp. 552-554 for its derivation)

N.B.: All quantities are referred to the stator

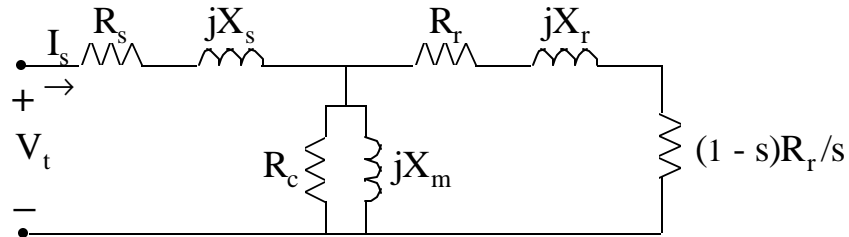


**Steinmetz Form I**

What's the meaning of this circuit's variables and elements?

$R_r/s = R_r + (1 - s)R_r/s$  so another form is (next slide) ...

Section 15.3: Development of Equivalent Circuits cont.

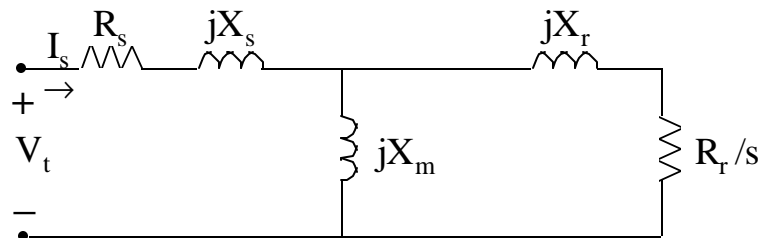


Steinmetz Form II

What's the meaning of this circuit's variables and elements?

For example, what's the meaning of  $R_r$  versus  $R_r/s$  (in Form I) and  $(1 - s)R_r/s$  (in Form II)?

Section 15.4: Performance Calculations



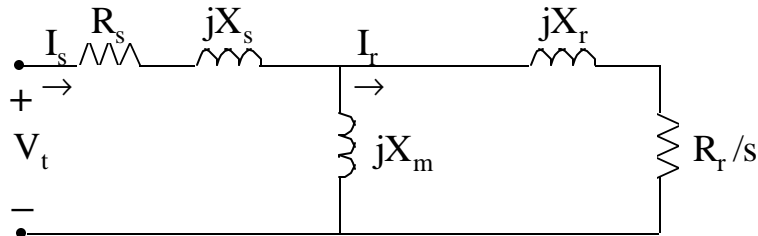
Steinmetz Form I

Next, various performance equations are developed from the Steinmetz circuits—with core losses neglected

Using the stator's line-to-neutral terminal voltage  $V_t$  as the reference phasor then:  $V_t = |V_t|/0^\circ$

And the stator's line current is:  $I_s = |I_s|/\theta$  so the average (per-phase) input power is:  $P_{in} = |V_t||I_s|\cos\theta$

Section 15.4: Performance Calculations cont.



Steinmetz Form I

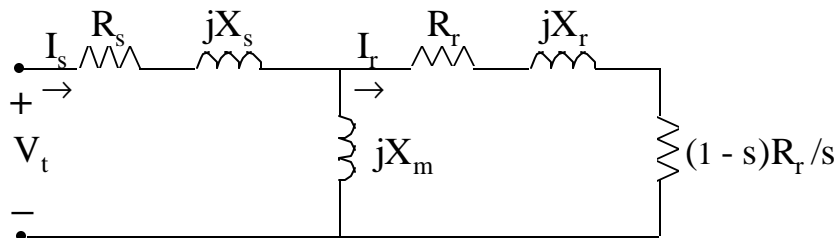
The stator copper losses are  $|I_s|^2 R_s$

The average power crossing the motor's air-gap (from the stator into the rotor) is:  $P_g = P_{in} - |I_s|^2 R_s$

All of  $P_g$  is dissipated in (the rotor's)  $R_r/s$  so  $P_g = |I_r|^2 R_r/s$

Which yields the rotor copper losses:  $|I_r|^2 R_r = sP_g$

Section 15.4: Performance Calculations cont.



Steinmetz Form II

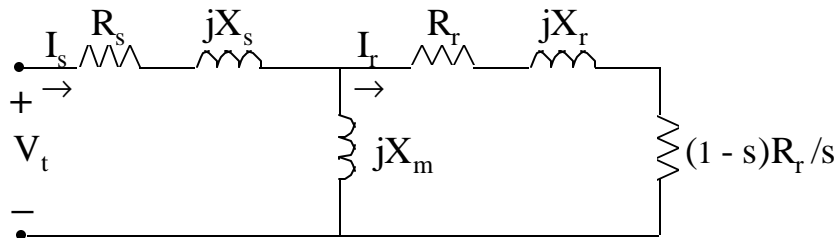
Subtracting the rotor's copper losses from the total rotor average power yields the average (per-phase) *developed* (electromagnetic) power:  $P_d = P_g - (|I_r|^2 R_r = sP_g)$

Ergo, or  $P_d = (1 - s)P_g$

What's  $P_d$ ?

It's the gross (per-phase) average *mechanical* power developed by the motor

Section 15.4: Performance Calculations cont.



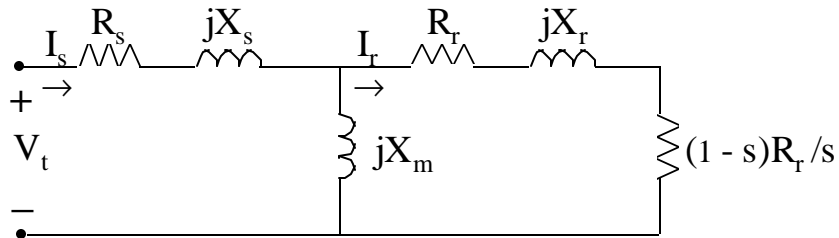
Steinmetz Form II

Note that  $P_d$  is dissipated by the resistor  $(1 - s)R_r/s$  so this resistor represents the motor's gross mechanical load

Subtracting the average (per-phase) mechanical windage and friction losses from the developed power yields the average (per-phase) mechanical output power:

$$P_{out} = P_d - P_{w\&f}$$

Section 15.4: Performance Calculations cont.



Steinmetz Form II

The electromagnetic (developed) torque  $T_e$  of the motor is its *total* developed (mechanical) power  $3P_d$  (for all *three* phases) divided by the motor's shaft speed  $\omega_m$

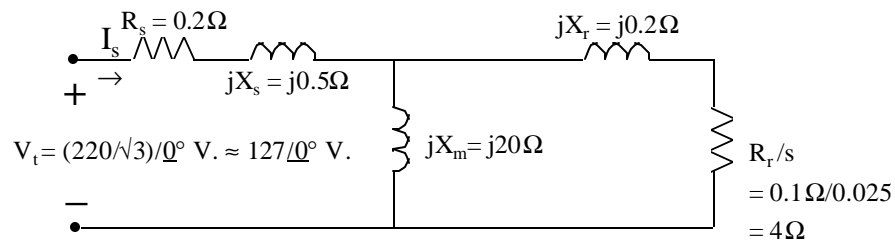
$\therefore T_e = 3P_d / \omega_m$ . However,  $\omega_m = (1 - s)\omega_s$ , where  $\omega_s$  is the **synchronous speed** and  $P_d = (1 - s)P_g$

Ergo,  $T_e = 3P_d / \omega_m = 3(1 - s)P_g / (1 - s)\omega_s \therefore T_e = 3P_g / \omega_s$

## Section 15.4: Performance Calculations cont.

### Example 15.2

The total iron and mechanical losses for a 220 V. three-phase induction motor (see per-phase model below) 350 W. For a slip of 2.5%, calculate the input current's magnitude, average output power, output torque and efficiency



Steinmetz Form I

12:23 PM Ohio University's Russ College of Engineering & Technology 17

## Section 15.4: Performance Calculations cont.

### Example 15.2 cont.

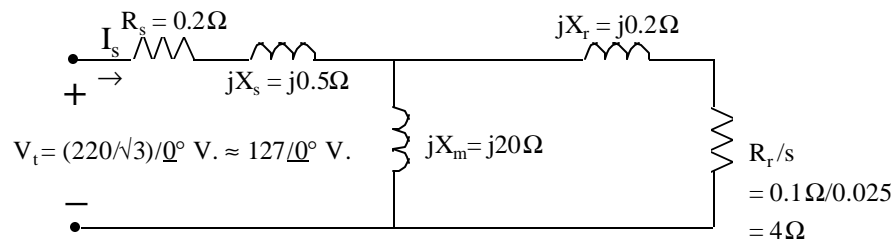
The (Thevenin) equivalent circuit impedance of the motor is:

$$Z_{Th} = (R_s + jX_s) + jX_m \parallel (R_r/s + jX_r) \quad \therefore$$

$$Z_{Th} = (0.2 + j0.5)\Omega + j20\Omega \parallel (0.2 + j4)\Omega \approx 4.23/+20^\circ\Omega \text{ and}$$

$$I_s = V_t / Z_{Th} = 127/0^\circ \text{ V.} / 4.23/+20^\circ\Omega \approx 30/-20^\circ \text{ A.}$$

$$\text{so } |I_s| = 30 \text{ A. and } P_{in} = 127\text{V.} \times 30\text{A.} \cos(-20^\circ) \approx 3.58 \text{ KW}$$



Steinmetz Form I

12:23 PM Ohio University's Russ College of Engineering & Technology 18

## Section 15.4: Performance Calculations cont.

### Example 15.2 cont.

Then the air-gap power is:

$$P_g = P_{in} - |I_s|^2 R_s = 3.58 \text{ KW} - (30\text{A.})^2 \times 0.2\Omega = 3.4 \text{ KW}$$

The developed power is:

$$P_d = (1 - s)P_g = (1 - 0.025) \times 3.4 \text{ KW} = 3.315 \text{ KW (per phase)}$$

The *per-phase* average output power is:

$$P_{out} = P_d - P_{loss} = 3.315 \text{ KW} - 350\text{W./3} = 3.198 \text{ KW (per phase)}$$

$$n = (1 - s)n_s = (1 - 0.025) \times (120 \times 60\text{Hz}/4\text{poles}) = 0.975 \times 1800\text{RPM} = 1755 \text{ RPM}$$

$$\therefore \omega_m = 1755 \text{ RPM} \times (2\pi \text{ rad./rev.}) \times (1 \text{ min.} / 60 \text{ sec.}) \approx 184 \text{ rad./sec}$$

Ergo, the output torque is:

$$T_{out} = 3P_{out}/\omega_m = 3 \times (3.198 \text{ KW}) / (184 \text{ rad./sec.}) \approx 52 \text{ Nt}\cdot\text{m}$$

And the efficiency is:

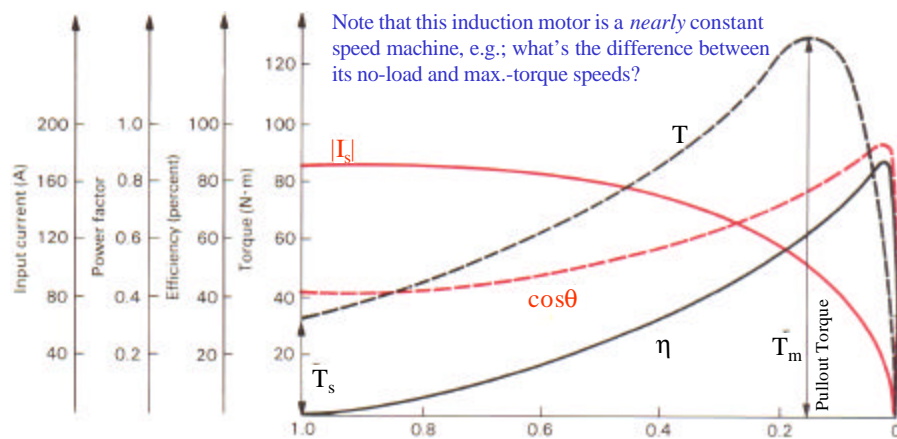
$$P_{out}/P_{in} \times 100\% = 3.198 \text{ KW} / 3.58 \text{ KW} \times 100\% \approx 89.3\%$$

12:23 PM Ohio University's Russ College of Engineering & Technology 19

## Section 15.4: Performance Calculations cont.

### Example 15.2 cont.

Repeating these calculations over a range of slips yields the figure below



12:23 PM Ohio University's Russ College of Engineering & Technology 20

## Section 15.4: Performance Calculations cont. Example 15.3

A two-pole three-phase 60 Hz. induction motor develops 25 KW (about 33.5 hp) of electromagnetic power at a certain speed. The rotational mechanical loss at this speed is 400 W. (0.54 hp). If the power crossing the air gap is 27 KW (36 hp), calculate the slip and output torque.

$$P_d = 25 \text{ KW} = (1 - s) \times 27 \text{ KW} = (1 - s)P_g$$

$$s = 1 - 25/27 \approx 0.074 \text{ (7.4\%)}$$

The synchronous speed is:

$$n_s = 120f/p = 120 \times 60 \text{ Hz.} / 2 \text{ poles} = 3600 \text{ RPM (377 rad./sec.)}$$

The shaft speed is  $n = (1 - s)n_s = 3334 \text{ RPM (349 rad./sec.)}$

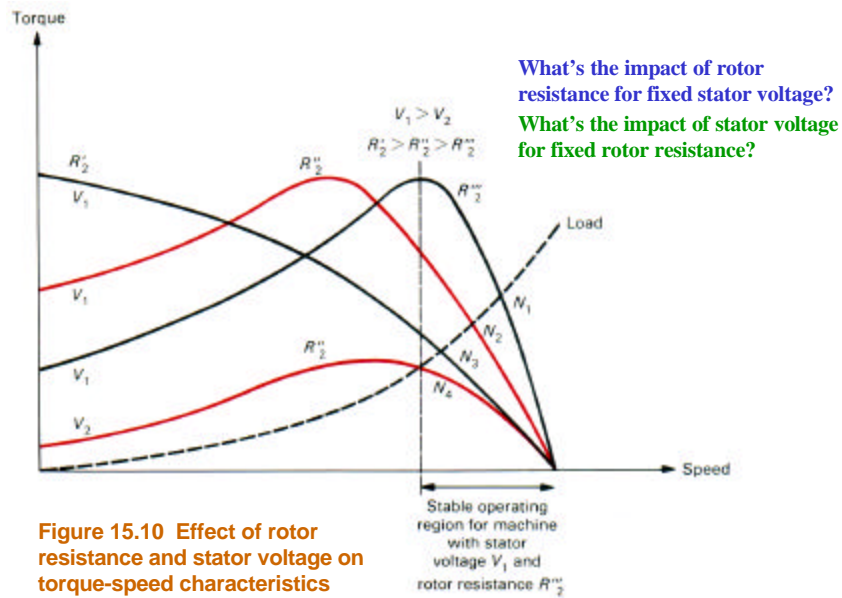
$$T_e = 3P_d/\omega_m = 25 \text{ KW} / 349 \text{ rad./sec.} = 71.63 \text{ N}\cdot\text{m}$$

Likewise,  $T_{w\&f} = P_{w\&f}/\omega_m = 400 \text{ W} / 349 \text{ rad./sec.} = 1.15 \text{ N}\cdot\text{m}$

$$\therefore T_{out} = T_e - T_{w\&f} = (71.63 - 1.15)\text{N}\cdot\text{m} = 70.48 \text{ N}\cdot\text{m}$$

12:23 PM *Ohio University's Russ College of Engineering & Technology* 21

## Section 15.5: Performance Criteria of Induction Motors



12:23 PM *Ohio University's Russ College of Engineering & Technology* 22

## Section 15.6: Speed and Torque Control of Induction Motors

Recall that induction motors are nearly constant (i.e., synchronous) speed machines

The synchronous speed is  $n_s = 120f/p$

Ergo, the motor's speed can be controlled by controlling the stator voltage's frequency (by way of power electronics)

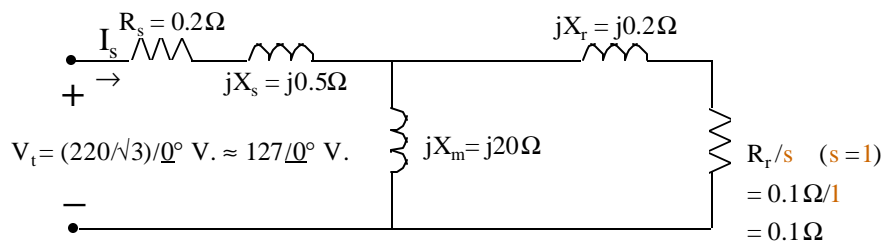
For further details see text pp. 561-567 for a (mostly) qualitative description of torque-speed control

## Section 15.7: Starting of Induction Motors

Recall that induction motors are essentially *rotating transformers*

On starting ( $s = 1$ ), the rotor's impedance is small (see below) so the motor is essentially a transformer with its secondary winding short-circuited

Ergo, the induction motor's starting current is high (typically 5-7 times full-load current)

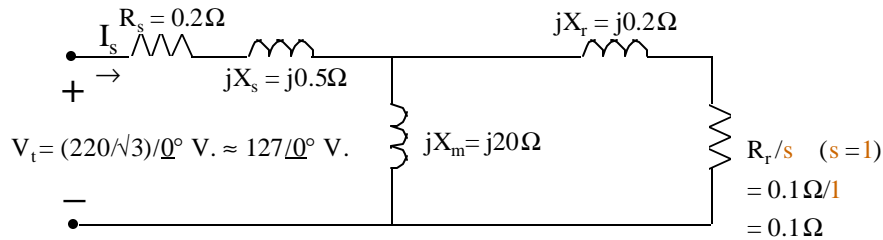


Example 15.2 on start ( $s=1$ )

## Section 15.7: Starting of Induction Motors cont.

The high starting current can be more detrimental to the motor's supply (mains) than the motor (e.g., supply overcurrent protection and voltage regulation problems)

For example, the low voltage ensuing from poor voltage regulation may prevent the motor from starting (i.e.,  $T \propto |V_t|^2$ ) and/or supply CBs may false-trip on start



Example 15.2 on start ( $s=1$ )

## Section 15.7: Starting of Induction Motors cont.

### Examples 15.4 and 15.5

An induction motor is designed to run at 5% slip on full load. If the motor draws 6 times its full-load current on starting (at rated voltage), estimate the ratio of the starting torque to the full-load torque. Repeat for a starting current of 3 times rated full-load current.

Key assumption:  $|I_s| \approx |I_r|$ , which is reasonable since the (rotating) transformer's  $|jX_m||R_c| \gg |R_r/s + jX_r|$

The full-load ( $s = 0.05$ ) torque is:

$$T_{FL} = P_{g,FL}/\omega_s = (|I_{r,FL}|^2 R_r / s_{FL}) / \omega_s \approx (|I_{s,FL}|^2 R_r / 0.05) / \omega_s \quad (\text{I.})$$

The starting ( $s = 1$ ) torque is:

$$T_{ST} = P_{g,ST}/\omega_s = (|I_{r,ST}|^2 R_r / s_{ST}) / \omega_s \approx (|I_{s,ST}|^2 R_r / 1) / \omega_s \quad (\text{II.})$$

$$(\text{II.}) / (\text{I.}) \text{ yields: } T_{ST} / T_{FL} \approx s_{FL} (|I_{s,ST}| / |I_{s,FL}|)^2 \quad (\text{III.})$$

Which for  $|I_{s,ST}| / |I_{s,FL}| = 6$  and  $|I_{s,ST}| / |I_{s,FL}| = 3$  yields:

$$T_{ST} / T_{FL} = 1.8 \text{ and } 0.45 \text{ respectively (Note: } 1.8 = 4 \times 0.45)$$

## Section 15.7: Starting of Induction Motors cont. Starting Techniques Based on Rotor-resistance and Reduced Stator Voltage

### Rotor Resistance Techniques

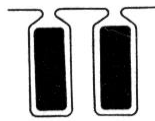
As suggested by text Figure 15.10 (p. 560), increased rotor resistance yields higher torque at lower speed

Inserting (exogenous) resistance into the rotor (while starting) is straight-forward for wound-rotor motors since the rotor circuit is electrically accessible

In cage (rotor)-type machines, the *skin effect* phenomenon is used to automatically insert resistance in the rotor (on starting) as follows (next slide)

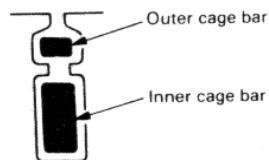
12:23 PM Ohio University's Russ College of Engineering & Technology 27

## Section 15.7: Starting of Induction Motors cont. Starting Techniques Based on Rotor-resistance and Reduced Stator Voltage cont.



Deep-bar rotor scheme

*Deep-bar* rotor depth is 2-3 times its width. At low slip (starting), the rotor-current frequency is high (60 Hz.) and as a result of skin effect, the rotor's AC currents crowd towards the tops of the bars (rotor's periphery) and thus flow through a smaller cross-sectional area ( $\therefore$  higher resistance) than they do at high slip (when  $f_r$  is a few Hz.).



Double-cage rotor scheme

*Double-cage* rotor employs two sets of rotor bars (one deeper than the other) to employ skin effect in a fashion similar to that of the deep-bar rotor scheme

12:23 PM Ohio University's Russ College of Engineering & Technology 28

## Section 15.7: Starting of Induction Motors cont. Example 15.6

An induction motor employs a wye-delta starter, *reduced-voltage starting scheme*, which connects the motor phases in wye on starting and in delta when the motor is running. The full-load slip is 4%, and the motor draws 9 times the full-load current if started directly (in delta) from the mains. Determine the ratio of starting torque to full-load torque.

Key assumption:  $|I_s| \approx |I_r|$ , which is reasonable since the (rotating) transformer's  $|jX_m||R_c| \gg |R_r/s + jX_r|$

Then, from Examples 15.4 and 15.5:  $T_{ST} / T_{FL} \approx s_{FL} (|I_{s,ST}| / |I_{s,FL}|)^2$

If the motor is delta-started,  $|I_{s,ST}| / |I_{s,FL}| = 9$  so that

$$T_{ST} / T_{FL} \approx s_{FL} (9)^2 = 0.04 \times 81 = 3.24$$

However, if it's wye-started,  $|I_{s,FL}|$  is attenuated by  $\sqrt{3}$  because  $|V_t|$  is attenuated by  $\sqrt{3}$  (and  $|I_{s,FL}| \approx |V_t| / |Z|$ , where:  $Z = (R_s + R_r/s_{FL}) + j(X_s + X_r)$  is fixed)

$$\text{Ergo, } T_{ST} / T_{FL} \approx s_{FL} (|I_{s,ST}| / |I_{s,FL}| / \sqrt{3})^2 = 0.04 \times (9 / \sqrt{3})^2 = 1.08$$

12:23 PM *Ohio University's Russ College of Engineering & Technology* 29

**We welcome your  
questions with  
Enthusiasm!!**



**Brian Manhire, Ph.D.**  
Professor of Electrical  
Engineering

OHIO UNIVERSITY

School of Electrical Engineering  
& Computer Science      740-593-1579 phone  
Stocker Center              740-593-0007 fax  
Athens OH 45701-2979      bmanhire1@ohiou.edu

**School of Electrical Engineering  
and  
Computer Science**



**Ohio University**

*Russ College of Engineering & Technology*