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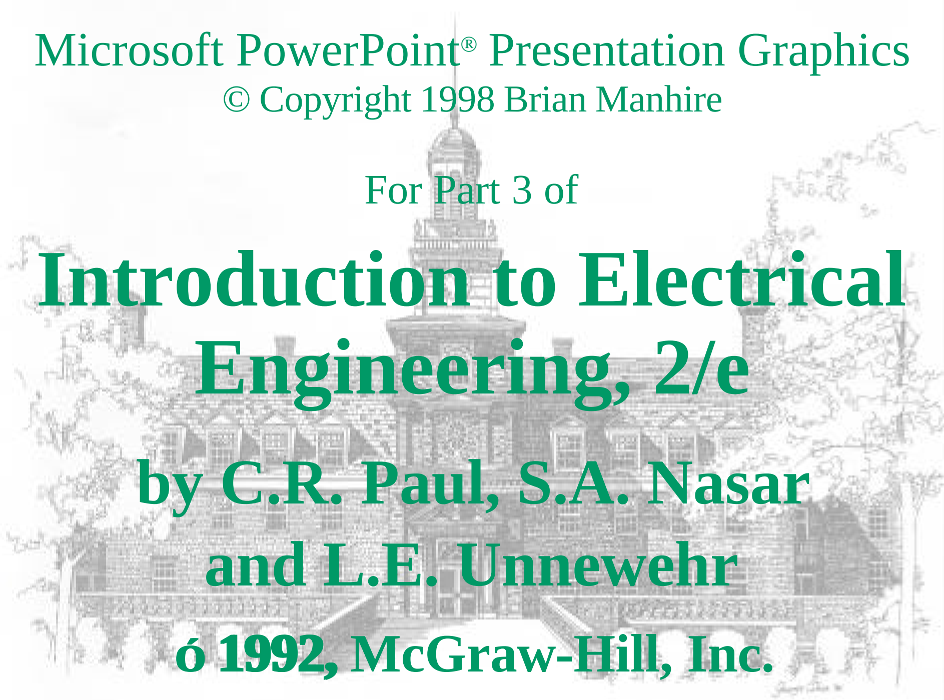
EE 315: Basic Electrical Engineering III

Prepared by Brian Manhire, Ph.D.
Professor of Electrical Engineering

Stocker Center, home of Ohio University's Russ College of Engineering & Technology



Microsoft PowerPoint® Presentation Graphics

For Part 3 of

Introduction to Electrical Engineering, 2/e

by C.R. Paul, S.A. Nasar and L.E. Unnewehr

Chapter 11: Polyphase Circuits

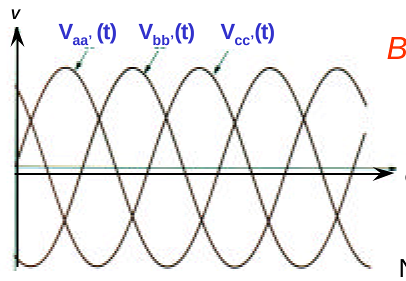
11.1 Three-Phase Systems

11.2 Three-Phase Connections

11.3 Power in Three-Phase Systems

11.4 Wye-Delta Equivalence

Section 11.1: Three-Phase Systems



Balanced set of 3 voltages, i.e.;

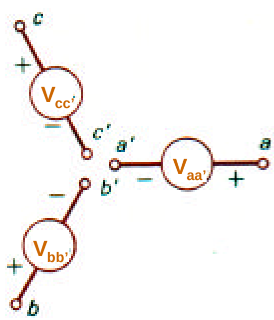
$V_{aa'}(t) + V_{bb'}(t) + V_{cc'}(t) = 0$, where:

$V_{aa'}(t) = V_m \sin(\omega t)$

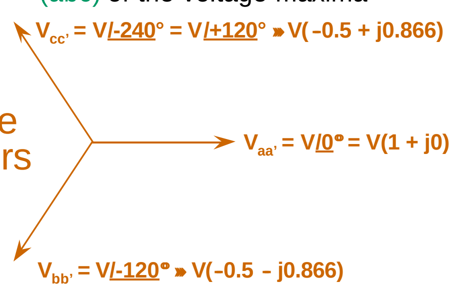
$V_{bb'}(t) = V_m \sin(\omega t - 120^\circ)$

$V_{cc'}(t) = V_m \sin(\omega t - 240^\circ)$

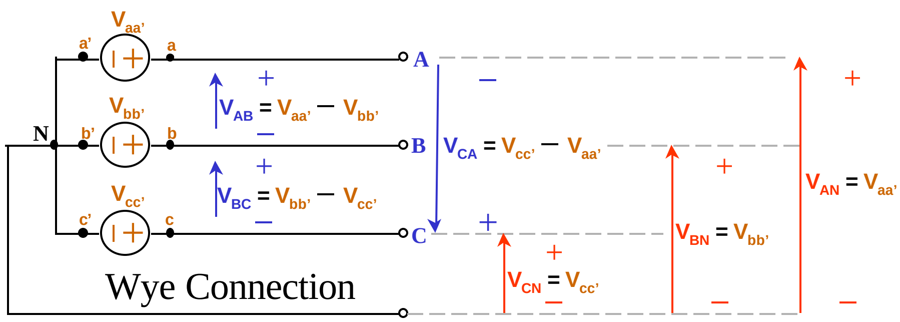
Note the *positive* (phase) *sequence* (abc) of the voltage maxima

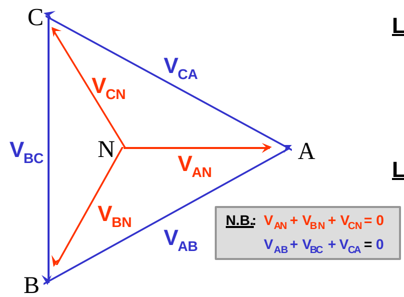


Voltage Phasors



Section 11.2: Three-Phase Connections





Line-to-Neutral Voltages = Phase Voltages

$V_{AN} = V_{aa'} = V\underline{/0°} = V(1 + j0)$

$V_{BN} = V_{bb'} = V\underline{/-120°} \gg V(-0.5 - j0.866)$

$V_{CN} = V_{cc'} = V\underline{/-240°} = V\underline{/+120°} \gg V(-0.5 + j0.866)$

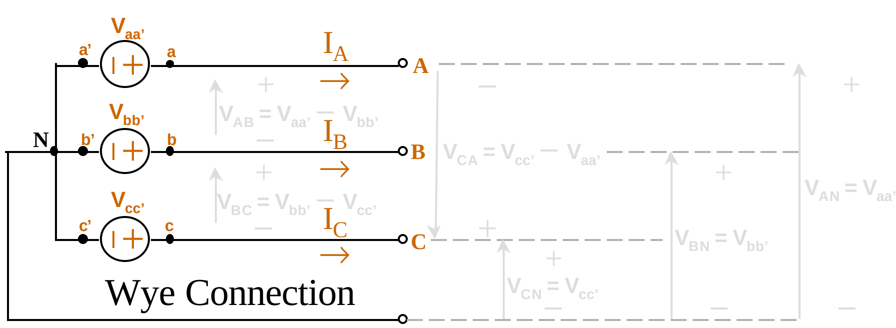
Line-to-Line Voltages:

$V_{AB} = \sqrt{3}V\underline{/+30°} \gg \sqrt{3}V(+0.866 + j0.5)$

$V_{BC} = \sqrt{3}V\underline{/-90°} \gg \sqrt{3}V(\ 0.0 \quad - j1.0)$

$V_{CA} = \sqrt{3}V\underline{/+150°} \gg \sqrt{3}V(-0.866 + j0.5)$

Section 11.2: Three-Phase Connections cont.



Line Currents I_A, I_B and I_C
= Phase (Source) Currents

Line-to-Neutral Voltages = Phase Voltages

$V_{AN} = V_{aa'} = V\underline{/0°} = V(1 + j0)$

$V_{BN} = V_{bb'} = V\underline{/-120°} \gg V(-0.5 - j0.866)$

$V_{CN} = V_{cc'} = V\underline{/-240°} = V\underline{/+120°} \gg V(-0.5 + j0.866)$

Line-to-Line Voltages:

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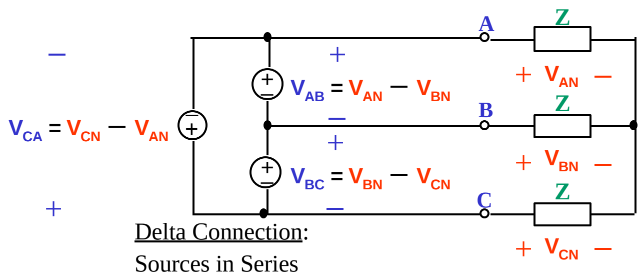
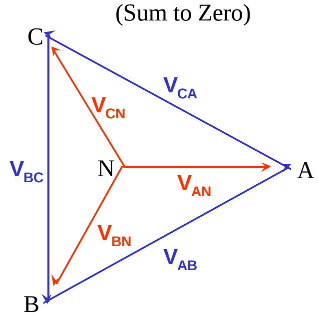
$V_{CA} = \sqrt{3}V\underline{/+150°} \gg \sqrt{3}V(-0.866 + j0.5)$

Section 11.2: Three-Phase Connections cont.



Balanced Wye Connection: Equal load Impedances (Z) connected to a common (neutral) point (N)

Delta Connection: Sources in Series (Sum to Zero)



Line-to-Line Voltages:

$V_{AB} = \sqrt{3}V\underline{/+30^\circ} \approx \sqrt{3}V(+0.866 - j0.5)$

$V_{BC} = \sqrt{3}V\underline{/-90^\circ} \approx \sqrt{3}V(+0.0 - j1.0)$

$V_{CA} = \sqrt{3}V\underline{/+150^\circ} \approx \sqrt{3}V(-0.866 + j0.5)$

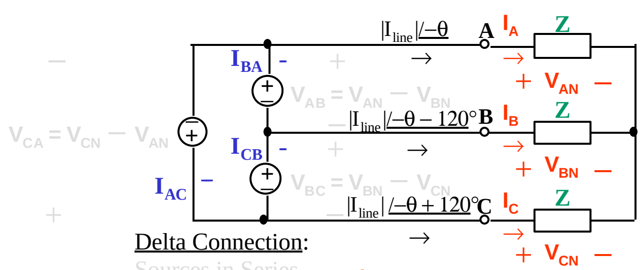
Line-to-Neutral Voltages = Phase Voltages

$V_{AN} = V\underline{/0^\circ} = V(1 + j0)$

$V_{BN} = V\underline{/-120^\circ} \approx V(-0.5 - j0.866)$

$V_{CN} = V\underline{/-240^\circ} = V\underline{/+120^\circ} \approx V(-0.5 + j0.866)$

Section 11.2: Three-Phase Connections cont.



Balanced Wye Connection: Equal load Impedances (Z) connected to a common (neutral) point (N)

Delta Connection: Sources in Series

Line Currents:

$I_A = V_{AN} / Z = |V_{AN}|\underline{/0^\circ} / |Z|\underline{/\theta} = |I_{line}|\underline{/-\theta}$ $(= I_{BA} - I_{AC})$

$I_B = V_{BN} / Z = |V_{BN}|\underline{/-120^\circ} / |Z|\underline{/\theta} = |I_{line}|\underline{/-\theta - 120^\circ}$ $(= I_{CB} - I_{BA})$

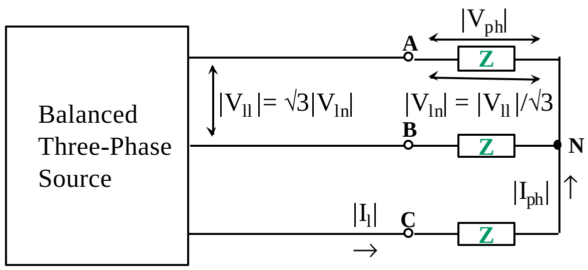
$I_C = V_{CN} / Z = |V_{CN}|\underline{/+120^\circ} / |Z|\underline{/\theta} = |I_{line}|\underline{/-\theta + 120^\circ}$ $(= I_{AC} - I_{CB})$

Delta's Phase Currents:

$I_{BA} = (|I_{line}|/\sqrt{3})\underline{/-\theta + 30^\circ}$, $I_{CB} = (|I_{line}|/\sqrt{3})\underline{/-\theta - 90^\circ}$ and $I_{AC} = (|I_{line}|/\sqrt{3})\underline{/-\theta + 150^\circ}$

Where: $I_A + I_B + I_C = 0$ and $I_{BA} + I_{CB} + I_{AC} = 0$ (i.e., both sets of three-phase currents are balanced.)

Section 11.2: Three-Phase Connections cont.



Balanced Wye Connection: Equal phase impedances (**Z**) connected to a common (neutral) point (N)

Balanced-Wye Voltage and Current *Magnitude* Relationships

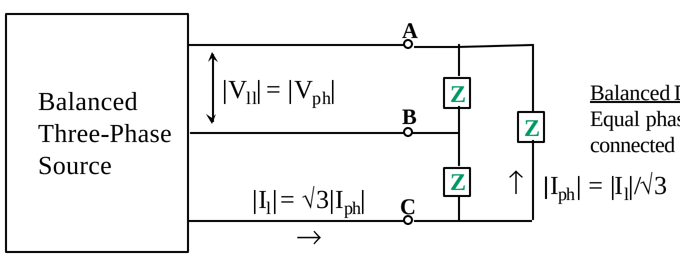
$|I_l|$= line current *magnitude* = phase current *magnitude* = $|I_{ph}|$

$$|V_{ll}| = \sqrt{3}|V_{ln}| \quad (= \sqrt{3}|V_{ph}|)$$

Three-Phase Apparent Power

$$|S_{3ph}| = 3|S_{ph}| = 3|V_{ph}||I_{ph}| = 3|V_{ln}||I_l| = 3(|V_{ll}|/\sqrt{3})|I_l| = \sqrt{3}|V_{ll}||I_l|$$

Section 11.2: Three-Phase Connections cont.



Balanced Delta Connection: Equal phase impedances (**Z**) connected series

Balanced-Delta Voltage and Current *Magnitude* Relationships

$|V_{ll}|$ = line-to-line voltage *magnitude* = phase voltage *magnitude* = $|V_{ph}|$

$$|I_l| = \sqrt{3}|I_{ph}|$$

Balanced Three-Phase Apparent Power

$$|S_{3ph}| = 3|S_{ph}| = 3|V_{ph}||I_{ph}| = 3|V_{ll}||I_{ph}| = 3|V_{ll}|(|I_l|/\sqrt{3}) = \sqrt{3}|V_{ll}||I_l|$$

(Same as Wye result)

Section 11.3: Power in Three-Phase Systems

As demonstrated in text section 11.3 (p. 461, equation 11.18), in balanced three-phase systems, the total three-phase instantaneous power is *constant*.

This is analogous to the total power output of an automobile's multi-cylinder ICE (i.e., what would happen if all cylinders fired at the same time?)

<u>Complex Power</u>

$$\mathbf{S_{3ph}} = 3\mathbf{S_{ph}} = 3\mathbf{V_{ph}}(\mathbf{I_{ph}})^* = 3|\mathbf{V_{ph}}||\mathbf{I_{ph}}|(\cos\theta + j\sin\theta)$$

where θ is the power factor angle (e.g., **Z**'s angle)

But $3|V_{ph}||I_{ph}|$ is the three-phase apparent power $|\mathbf{S_{3ph}}| = \sqrt{3}|V_{ll}||I_l|$

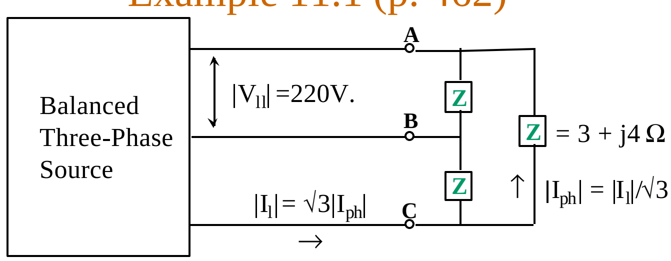
Ergo, $\mathbf{S_{3ph}} = \sqrt{3}|V_{ll}||I_l|(\cos\theta + j\sin\theta) = P_{3ph} + jQ_{3ph}$

$\therefore$ $P_{3ph} = |S_{3ph}|\cos\theta = \sqrt{3}|V_{ll}||I_l|\cos\theta$ and

$Q_{3ph} = |S_{3ph}|\sin\theta = \sqrt{3}|V_{ll}||I_l|\sin\theta$

Section 11.3: Power in Three-Phase Systems cont.

Example 11.1 (p. 462)



Calculate the magnitude of the line current and the total three-phase *complex* power supplied to the load.

$|I_{ph}| = |V_{ll}| / |\mathbf{Z}| = 220\text{ V.} / |3 + j4|\ \Omega = 220\text{ V.} / 5\ \Omega = 44\text{ A.}$

$|I_l| = \sqrt{3}|I_{ph}| = \sqrt{3} \times 44\text{ A.} \approx \underline{76.21\text{ A.}}$ (answer)

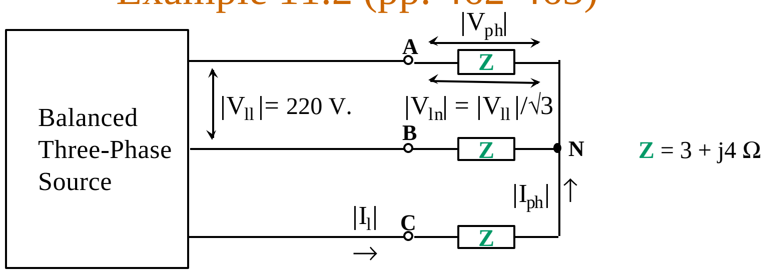
$\mathbf{Z} = 3 + j4\ \Omega \approx 5\underline{/\ 53.13^\circ}\ \Omega$ $\therefore$ $\theta \approx 53.13^\circ$ so that …

$S_{3ph} = \sqrt{3}|V_{ll}||I_l|(\cos\theta + j\sin\theta) = \sqrt{3} \times 220\text{ V.} \times 76.21\text{ A.} \times (0.6 + j0.8)$

$S_{3ph} \approx \underline{17.4\text{ KW} + j23.2\text{ KVAR}}$ (answer) $\approx 29\underline{/\ 53.13^\circ}$ KVA

Section 11.3: Power in Three-Phase Systems cont.

Example 11.2 (pp. 462-463)



Calculate the magnitude of the phase voltage and the total three-phase *complex* power supplied to the load.

$|V_{ph}| = |V_{ln}| = |V_{ll}|/\sqrt{3} = 220$ V. / $\sqrt{3} \approx$ <u>127 V.</u> (answer)

Z $= 3 + j4\ \Omega \approx 5\angle 53.13°\ \Omega$ $\therefore$ $\theta \approx 53.13°$ so that …

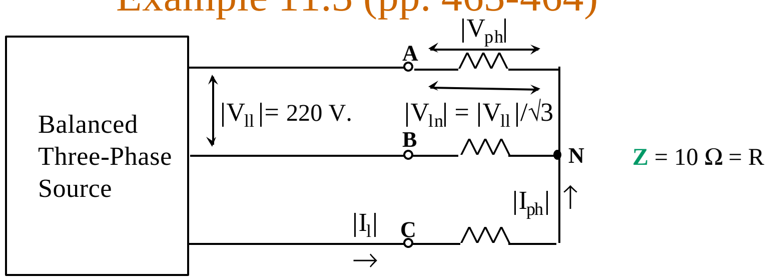
$|I_l| = |I_{ph}| = |V_{ln}|/|\mathbf{Z}| \approx 127$ V./ 5 Ω ≈ 25.4 A. so that ...

$S_{3ph} = \sqrt{3}|V_{ll}||I_l|(\cos\theta + j\sin\theta) = \sqrt{3} \times 220$ V. $\times$ 25.4 A. $\times (0.6 + j0.8)$

$S_{3ph} \approx$ <u>5.81 KW + j7.74 KVAR</u> (answer) $\approx 9.68\angle 53.13°$ KVA

Section 11.3: Power in Three-Phase Systems cont.

Example 11.3 (pp. 463-464)



Calculate the total three-phase power supplied to the load.

$|V_{ph}| = |V_{ln}| = |V_{ll}|/\sqrt{3} = 220$ V. / $\sqrt{3} \approx 127$ V.

Z $= 10 + j0\ \Omega = 10\angle 0°\ \Omega$ $\therefore$ $\theta = 0°$ so that …

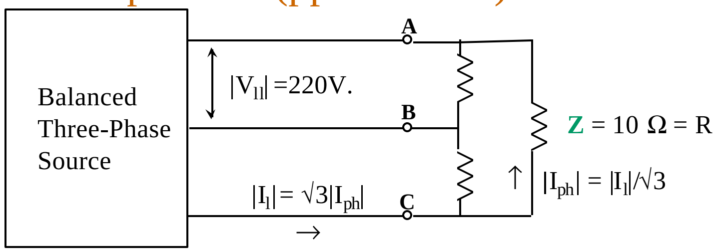
$|I_l| = |I_{ph}| = |V_{ln}|/|\mathbf{Z}| \approx 127$ V./ 10 Ω ≈ 12.7 A. so that ...

$S_{3ph} = \sqrt{3}|V_{ll}||I_l|(\cos\theta + j\sin\theta) = \sqrt{3} \times 220$ V. $\times$ 12.7 A. $\times (1 + j0)$

$S_{3ph} \approx 4.84$ KW + j0 KVAR $\therefore$ $P_{3ph} \approx$ <u>4.84 KW</u> (answer)

Section 11.3: Power in Three-Phase Systems cont.

Example 11.3 (pp. 463-464) cont.



Calculate the total three-phase power supplied to the load.

$|I_{ph}| = |V_{ll}| / |\mathbf{Z}| = 220$ V. / 10 Ω = 220 V. / 10 Ω = 22 A.

$|I_l| = \sqrt{3}|I_{ph}| = \sqrt{3} \times 22$ A. ≈ <u>38.12 A.</u> (answer)

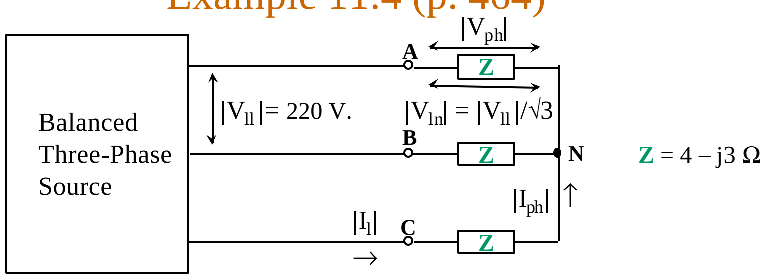
$\mathbf{Z} = 10 + j0\ \Omega = 10\angle 0°\ \Omega \quad \therefore\ \theta \approx 0°$ so that …

$S_{3ph} = \sqrt{3}|V_{ll}||I_l|(\cos\theta + j\sin\theta) = \sqrt{3} \times 220\text{ V.} \times 38.12\text{ A.} \times (1 + j0)$

$S_{3ph} \approx 14.52$ KW + j0 KVAR $\quad \therefore\ P_{3ph} \approx$ <u>14.52 KW</u> (answer)

Section 11.3: Power in Three-Phase Systems cont.

Example 11.4 (p. 464)



Calculate the line-to neutral voltage phasors, the line-to-line voltage phasors, the line current phasors and the load's three-phase complex power given V_{AN} is the reference phasor and the phase sequence is abc.

$V_{AN} = (220\text{ V.}/\sqrt{3})\angle 0° \approx 127\angle 0°$ V. (Ref. Phasor) $\therefore\ V_{BN} = 127\angle -120°$ V.

and $V_{CN} = 127\angle +120°$ V. (for abc sequence) and from the voltage "triangle,"

$V_{AB} = 220\angle +30°$ V., $V_{BC} = 220\angle -90°$ V. and $V_{CA} = 220\angle +150°$ V.

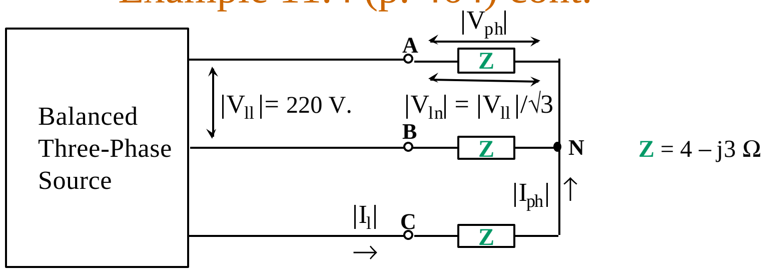
The line-current phasors are $I_A = V_{AN}/\mathbf{Z} = 127\angle 0°\text{ V.}/(4 - j3)\ \Omega$ or

$I_A = 127\angle 0°\text{ V.}/5\angle -36.87°\Omega \approx 25.4\angle +36.87°$ A. Similarly …

$I_B = 25.4\angle +83.13°$ A. and $I_C = 25.4\angle +156.87°$ A.

Section 11.3: Power in Three-Phase Systems cont.

Example 11.4 (p. 464) cont.

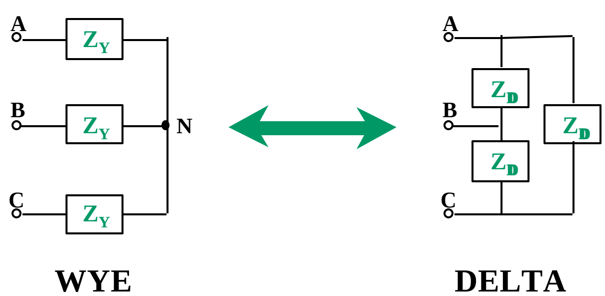


The load's three-phase complex power is:

$S_{3ph} = \sqrt{3}|V_{ll}||I_l|(\cos\theta + j\sin\theta) = \sqrt{3} \times 220\text{ V.} \times 25.4\text{ A.} \times (0.8 - j0.6)$

$S_{3ph} \approx$ <u>7.74 KW – j5.81 KVAR</u> (answer) $\approx 9.68\underline{/-36.87°}$ KVA

Section 11.4 Wye-Delta Equivalence

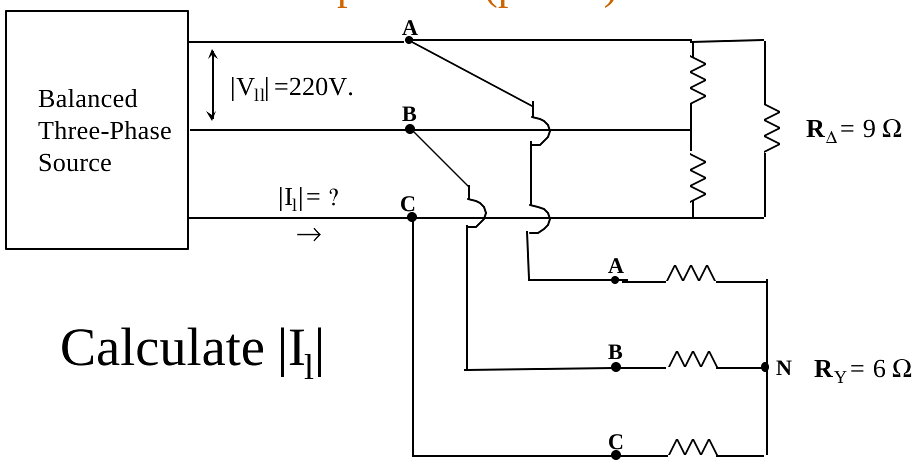


$$Z_D = 3Z_Y$$

(See text p. 465 for derivation)

Section 11.4 Wye-Delta Equivalence cont.

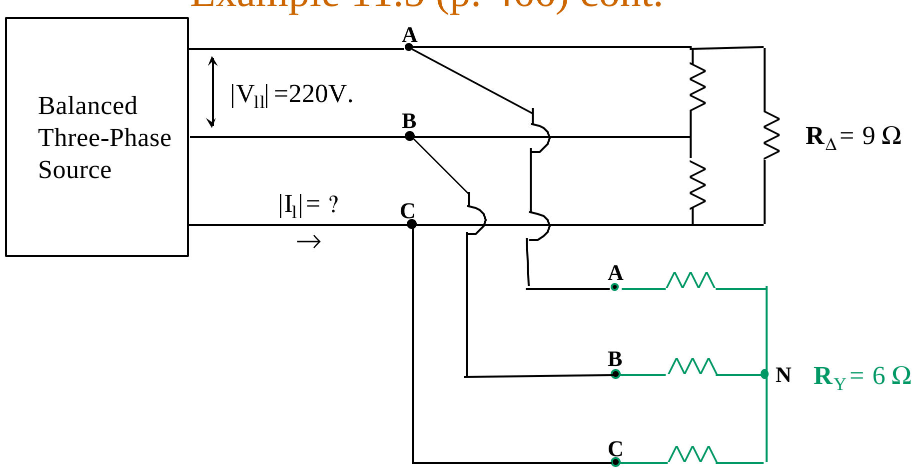
Example 11.5 (p. 466)



Calculate $|I_l|$

Section 11.4 Wye-Delta Equivalence cont.

Example 11.5 (p. 466) cont.

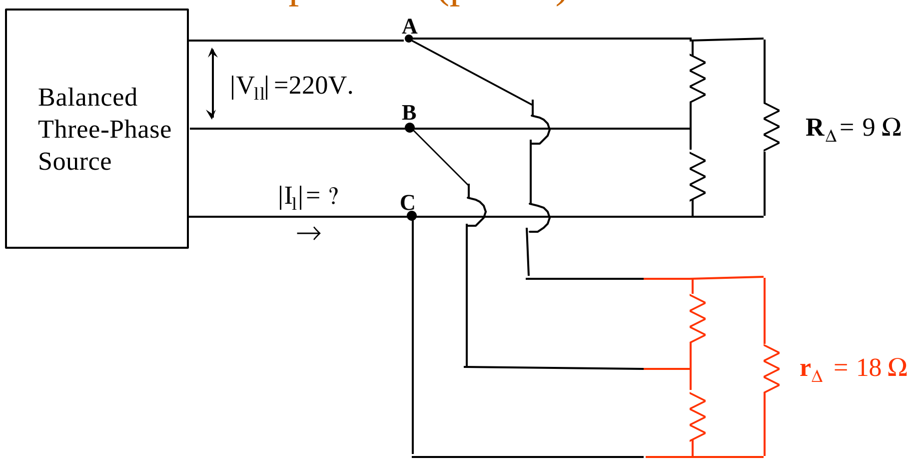


Step 1: Convert $\mathbf{R}_Y = 6\ \Omega$ to its equivalent delta:

$$r_\Delta = 3\mathbf{R}_Y = 18\ \Omega$$

Section 11.4 Wye-Delta Equivalence cont.

Example 11.5 (p. 466) cont.

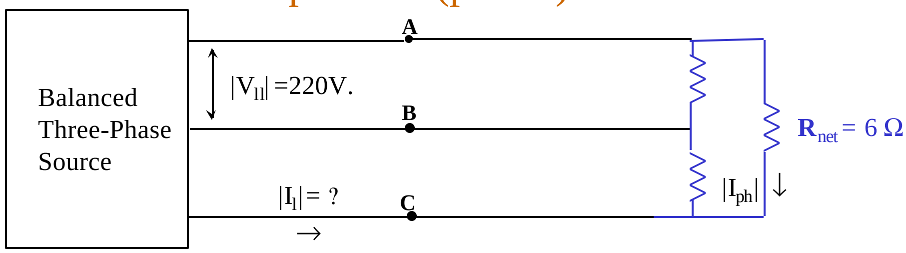


Step 2: Combine parallel (delta) resistors:

$R_{net} = \mathbf{R}_{\Delta} || \mathbf{r}_{\Delta} = 6\ \Omega$

Section 11.4 Wye-Delta Equivalence cont.

Example 11.5 (p. 466) cont.



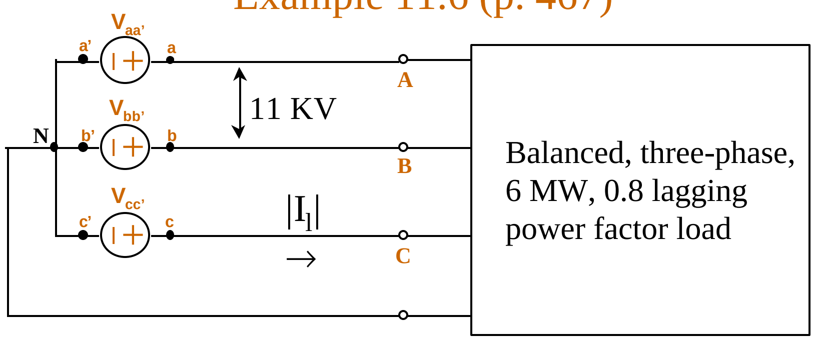
Step 3: $|I_{ph}| = |V_{ll}|/R_{net} = 220$ V./ $6\ \Omega \approx 36.67$ A.

$\therefore\ |I_l| = \sqrt{3}|I_{ph}| \approx 63.51$ A.

(See text pp. 466-467 for an alternate approach)

Section 11.4 Wye-Delta Equivalence cont.

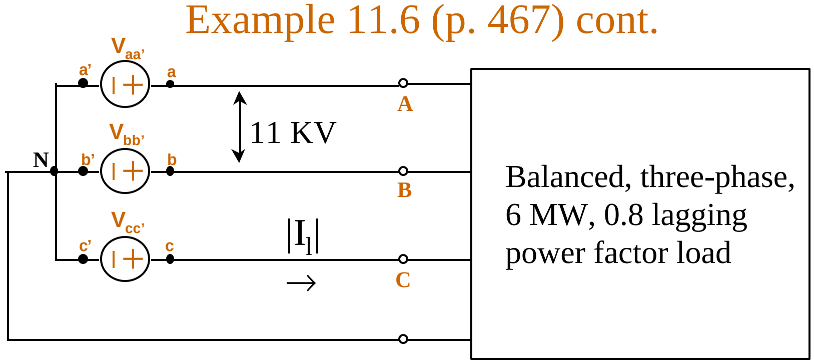
Example 11.6 (p. 467)



Find the total reactive power drawn by the load and the magnitude of the generator's line current ($|I_l|$)

Section 11.4 Wye-Delta Equivalence cont.

Example 11.6 (p. 467) cont.



$P_{3ph} = 6\text{ MW} = \sqrt{3} \times 11\text{ KV} \times |I_l| \times 0.8 \quad (= \sqrt{3}|V_{ll}|\,|I_l|\cos\theta)$

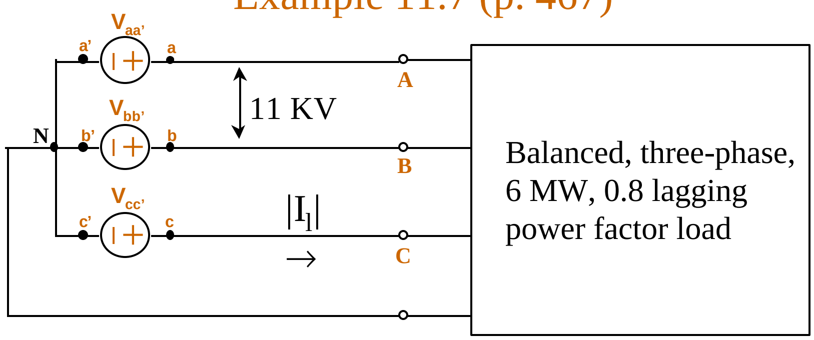
$\therefore \quad |I_l| \approx 393.6$ A. Then

$Q_{3ph} = \sqrt{3}|V_{ll}|\,|I_l|\sin\theta = \sqrt{3} \times 11\text{ KV} \times 393.6\text{ A.} \times 0.6 \approx +4.5\text{ MVAR}$

(Note that $Q_{3ph} > 0$ since the power factor is lagging)

Section 11.4 Wye-Delta Equivalence cont.

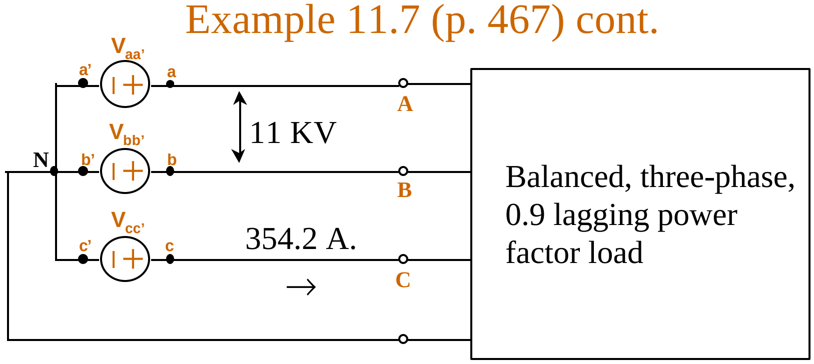
Example 11.7 (p. 467)



Find the total average power drawn by the load if the power factor of Example 11.6's load (shown above) is changed to 0.9 lagging and its line current is also reduced by 10% to 354.2 A.

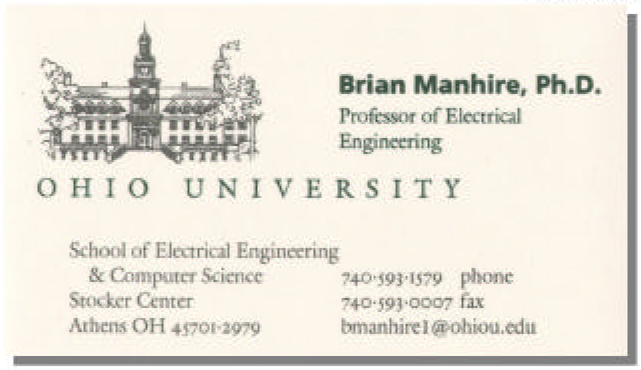
Section 11.4 Wye-Delta Equivalence cont.

Example 11.7 (p. 467) cont.



$$
\begin{aligned}
P_{3ph} &= \sqrt{3}|V_{ll}|\,|I_l|\cos\theta \\
&= \sqrt{3} \times 11\text{ KV} \times 354.2\text{ A.} \times 0.9 \\
&= 6.074\text{ MW}
\end{aligned}
$$

We welcome your questions with Enthusiasm!!



Brian Manhire, Ph.D.
Professor of Electrical Engineering

OHIO UNIVERSITY

School of Electrical Engineering
& Computer Science
Stocker Center
Athens OH 45701-2979

740-593-1579 phone
740-593-0007 fax
bmanhire1@ohiou.edu

School of Electrical Engineering and Computer Science

Ohio University

Russ College of Engineering & Technology