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EE 313: Basic Electrical Engineering I

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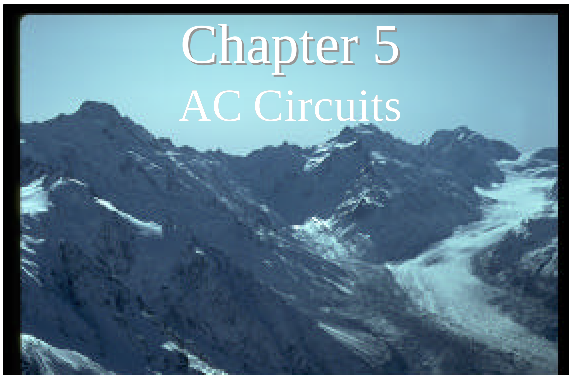
For Part 1 of

Introduction to Electrical Engineering, 2/e

by C.R. Paul, S.A. Nasar and L.E. Unnewehr



Introduction to Electrical Engineering

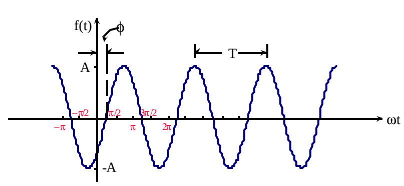
Chapter 5: AC Circuits

5.1 Sinusoidal (AC) Sources
5.2 Response of Circuits to AC Sources
5.3 Complex Numbers and Complex Algebra
5.4 Use of the Complex Exponential Source
5.5 The Phasor Circuit
5.6 Average Power, Reactive Power and Power Factor
5.7 Frequency Response of Circuits

5.1 Sinusoidal (AC) Sources

$f(t) = A\sin(\omega t + \phi)$

where A = Amplitude, ω = Radian Frequency (Rad./Sec.),
f = Frequency (Hz., formerly cps) where $\omega = 2\pi f$,
T = 1/f = Period (in Sec., mSec., etc.) and
ϕ = Phase angle (in Rad. or Degrees) where f(t) is said to "Lag" (time delay) its $\phi = 0$ counterpart if $\phi < 0$ and "lead" (time advance) if $\phi > 0$



5.1 Sinusoidal (AC) Sources cont.

Some useful identities

$A\cos(\omega t) = A\sin(\omega t + \pi/2) = A\sin(\omega t + 90°)$

$A\sin(\omega t) = A\cos(\omega t - \pi/2) = A\cos(\omega t - 90°)$

$-A\cos(\omega t) = A\cos(\omega t \pm \pi) = A\cos(\omega t \pm 180°)$

$-A\sin(\omega t) = A\sin(\omega t \pm \pi) = A\sin(\omega t \pm 180°)$

$\sin(\omega t \pm \phi) = \sin(\omega t)\cos(\phi) \pm \cos(\omega t)\sin(\phi)$

$\cos(\omega t + \phi) = \cos(\omega t)\cos(\phi) - \sin(\omega t)\sin(\phi)$

$\cos(\omega t - \phi) = \cos(\omega t)\cos(\phi) + \sin(\omega t)\sin(\phi)$

$A\sin(\omega t + \phi) = A_1\sin\omega t + A_2\cos\omega t$
where: $A_1 = A\cos\phi$ and $A_2 = A\sin\phi$

$B\cos(\omega t + \phi) = B_1\cos\omega t + B_2\sin(\omega t)$
where: $B_1 = B\cos\phi$ and $B_2 = -B\sin\phi$

5.2 Steady-State Response of Circuits to AC Sources

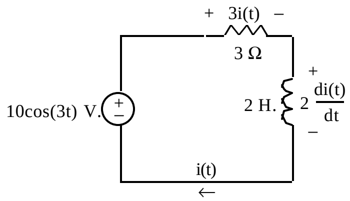
KVL for the RL circuit below yields the differential equation:

$$2(di(t)/dt) + 3i(t) = 10\cos(3t)$$

The mathematical objective is to find a *particular solution* of the equation (i.e., *any* solution that will satisfy the equation)

The standard approach is to choose

$i(t) = I_C\cos 3t + I_S\sin 3t$ (an educated guess, but why?)



5.2 Steady-State Response of Circuits to AC Sources

Substituting $i(t) = I_C\cos 3t + I_S\sin 3t$ into the differential equation $2(di(t)/dt) + 3i(t) = 10\cos(3t)$ yields:

$2(-3I_C\sin 3t + 3I_S\cos 3t) + 3(I_C\cos 3t + I_S\sin 3t) = 10\cos(3t)$ or …

$(3I_C + 6I_S)\cos 3t + (-6I_C + 3I_S)\sin 3t = 10\cdot\cos 3t + 0\cdot\sin 3t$

Equating coefficients yields:

$3I_C + 6I_S = 10$ and $-6I_C + 3I_S = 0$

Which have solution $I_C \approx 0.667$ and $I_S \approx 1.33$

So a particular solution is $i(t) \approx 0.667\cdot\cos 3t + 1.33\cdot\sin 3t$

However, $A\cdot\cos(\omega t) + B\cdot\sin(\omega t) = (A^2 + B^2)^{1/2}\cos(\omega t - \tan^{-1}(B/A))$

$\therefore\ i(t) \approx 1.491\cos(3t - 63.43°)$

Moral: There's gotta be a better way!

5.3 Complex Numbers and Complex Arithmetic

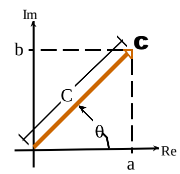
The *phasor method* (invented in 1893 by Charles Proteus Steinmetz, 1865-1923) is the backbone of AC steady-state circuit analysis

The principle benefit of the phasor method is that its concepts essentially render AC steady-state circuit analysis analogous to DC steady-state analysis—by way of complex arithmetic

Ergo, in order to gainfully employ the phasor method, the electric circuit analyst must be proficient in working with complex numbers, complex arithmetic and complex algebra

5.3 Complex Numbers and Complex Arithmetic cont.

A complex number manifests itself in two forms—and consists of two numbers (per form) and presents itself as a vector in the complex plane as shown below



$\mathbf{C} = a + jb$ (Rectangular form)

$\mathbf{C} = C\,e^{j\theta} = C\,\underline{/\theta}$ (Polar form)

where

$j = (-1)^{1/2}$, $a = \mathrm{Re}(\mathbf{C}) = C\cos\theta$, $b = \mathrm{Im}(\mathbf{C}) = C\sin\theta$

$C = (a^2 + b^2)^{1/2}$ and $\theta = \tan^{-1}(b/a)$

5.3 Complex Numbers and Complex Arithmetic cont.

Addition and subtraction of complex numbers is defined in rectangular form *only* as follows:

If $\mathbf{A} = a + jb$ and $\mathbf{B} = c + jd$ then

$\mathbf{A} + \mathbf{B} = (a + c) + j(b + d)$ and

$\mathbf{A} - \mathbf{B} = (a - c) + j(b - d)$

Multiplication and division is defined in polar form as follows:

If $\mathbf{A} = A\underline{/\theta_A}$ and $\mathbf{B} = B\underline{/\theta_B}$ then

$\mathbf{AB} = AB\underline{/\theta_A + \theta_B}$ and

$\mathbf{A/B} = A/B\underline{/\theta_A - \theta_B}$

and in rectangular form as follows ...

5.3 Complex Numbers and Complex Arithmetic cont.

Multiplication and division in rectangular form:

If $\mathbf{A} = a + jb$ and $\mathbf{B} = c + jd$ then

$\mathbf{AB} = (a + jb)(c + jd)$

$= (ac - bd) + j(ad + bc)$ and

$\mathbf{A/B} = (a + jb)/(c + jd)$

$= [(ac + bd) + j(bc - ad)]/(c^2 + d^2)$

Also, the *conjugate* of a phasor $\mathbf{A} = a + jb = C\underline{/\theta}$ is

$\mathbf{A}^* = a - jb = C\underline{/-\theta}$ so that $\mathbf{AA}^* = (a + jb)(a - jb)$

$= a^2 + b^2 = C^2$ also …

$1 = +1 + j0 = 1\underline{/0}$; $-1 = -1 + j0 = 1\underline{/\pm 180°}$

and $j = 0 + j1 = 1\underline{/90°}$

5.3 Complex Numbers and Complex Arithmetic cont.

Given: $\mathbf{A} = 1 - j3$ and $\mathbf{B} = 2\angle{-30^\circ}$, the following complex arithmetic yields the following numerical results (see text Problem 5.7, p. 202):

$\mathbf{A} + \mathbf{B} = (1 - j3) + 2\angle{-30^\circ} \approx (1 - j3) + (1.732 - j1)$
$\approx 2.732 - j4 \approx 4.844\angle{-55.67^\circ}$

$\mathbf{A} - \mathbf{B} = (1 - j3) - 2\angle{-30^\circ} \approx (1 - j3) - (1.732 - j1)$
$\approx -0.732 - j2 \approx 2.130\angle{-110.1^\circ}$

$\mathbf{AB} = (1 - j3)(2\angle{-30^\circ}) \approx (3.162\angle{-71.57^\circ})(2\angle{-30^\circ})$
$\approx 6.324\angle{-101.57^\circ} \approx -1.268 - j6.195$

$\mathbf{A/B} \approx (3.162\angle{-71.57^\circ})/(2\angle{-30^\circ}) \approx 1.581\angle{-41.57^\circ}$
$\approx 1.183 - j1.049$

$1/\mathbf{A} \approx 1\angle{0^\circ}/(3.162\angle{-71.57^\circ}) \approx 0.3163\angle{+71.57^\circ}$
$\approx 0.1 - j0.3$ and $\mathbf{AA}^* \approx 3.162^2 \approx 10.0$

5.4 Use of Complex Exponential Source

In AC steady-state circuit analysis, sinusoidal sources can be replaced by (mathematically) related complex exponential sources

Then the particular solution (steady-state response) for the exponential source(s) scenario is also a complex exponential that is *much* easier to determine than its sinusoidal-source counterpart—in fact, it can be found without (directly) using the time dependent portion of any source or response complex exponentials at all!

The particular solution for any sinusoidally driven (steady-state) scenario can be easily deduced from its complex exponential counterpart

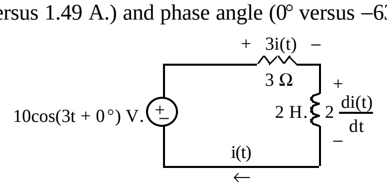
All of the above logic depends on linearity—so it's the first item on our "phasor analysis" agenda

5.4 Use of Complex Exponential Source cont.

However, before proceeding to linearity recall that for the circuit below the circuit's current response to the sinusoidal voltage source is $i(t) \approx 1.49\cos(3t - 63.43^\circ)$ A.

How do the voltage source and current response differ mathematically?

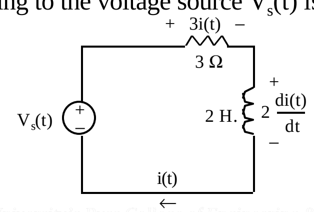
They differ by only two *numbers*—magnitude (10 V. versus 1.49 A.) and phase angle (0° versus –63.43°)!



5.4 Use of Complex Exponential Source cont.

Returning to linearity, consider the scenario shown below and analyzed earlier—where the original sinusoidal source 10cos(3t) V. (10sin(3t) V. in text Section 5.2) has been replaced by the generic source $V_s(t)$

The differential equation relating the current i(t) responding to the voltage source $V_s(t)$ is ...



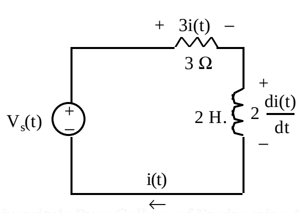
5.4 Use of Complex Exponential Source cont.

$2(di(t)/dt) + 3i(t) = V_s(t)$

Is this source-response relationship linear?

How can this question be answered?

Mathematically interrogate the differential equation to see if superposition applies!



5.4 Use of Complex Exponential Source cont.

Let $i_1(t)$ be a particular solution for source $V_{s1}(t)$ so that
$2(di_1(t)/dt) + 3i_1(t) = V_{s1}(t)$ (I.)

Let $i_2(t)$ be a particular solution for source $V_{s2}(t)$ so that
$2(di_2(t)/dt) + 3i_2(t) = V_{s2}(t)$ (II.)

Multiply (II.) by the constant K to obtain
$2(d[Ki_2(t)]/dt) + 3[Ki_2(t)] = KV_{s2}(t)$ (III.)

$\therefore$ $Ki_2(t)$ is a particular solution for source $KV_{s2}(t)$

Add equations (I.) and (III.) to obtain
$2(d[i_1(t) + Ki_2(t)]/dt) + 3[i_1(t) + Ki_2(t)] = [V_{s1}(t) + KV_{s2}(t)]$

$\therefore [i_1(t) + Ki_2(t)]$ is a particular solution for $[V_{s1}(t) + KV_{s2}(t)]$

$\therefore$ superposition applies ($\therefore$ the circuit is linear)

5.4 Use of Complex Exponential Source cont.

Next consider time-shifting the circuit's voltage source by the time delay τ

This is achieved mathematically by replacing (renaming) the *independent* variable t with $t - \tau$ to obtain …

$2(di(t - \tau)/dt) + 3i(t - \tau) = V_s(t - \tau)$

Which indicates that time-shifting the input by τ time-shifts the particular solution (output) by the same amount (τ)

So if $V_s(t) = 10\sin(3t)$ V. (= $10\cos(3t - \pi/2)$ V.), ...
What's i(t)?

$i(t) \approx 1.49\cos(3t - 63.43° - \pi/2)$ A. $\therefore$
$i(t) \approx 1.49\sin(3t - 63.43°)$ A.

(Which is verified on text p. 150)

5.4 Use of Complex Exponential Source cont.

Next consider the (Euler's Equation) ensemble of sources
$10\cos(3t) + j10\sin(3t)$ V. = $10e^{j3t}$ V.

Note that the above equation is just $[V_{s1}(t) + KV_{s2}(t)]$ with $V_{s1}(t) = 10\cos(3t)$, $V_{s2}(t) = 10\sin(3t)$ and K = j

Superposition applies so from earlier results

$i(t) = i_1(t) + ji_2(t)$

where $i_1(t) \approx 1.49\cos(3t - 63.43°)$ A.

and $i_2(t) \approx 1.49\sin(3t - 63.43°)$ A.

Then $i(t) \approx 1.49\cos(3t - 63.43°) + j1.49\sin(3t - 63.43°)$ A.

Which by Euler's Equation is $i(t) \approx 1.49e^{j(3t - 63.43°)}$ or …

$i(t) \approx 1.49e^{-j 63.43°}e^{j3t}$ where the complex coefficient of the e^{j3t} time function is the *complex number* **I » 1.49e$^{-j 63.43°}$**

5.4 Use of Complex Exponential Source cont.

Recapitulation

If the source is $\mathbf{V_s(t)} = 10e^{j3t}$ V. ,
the response is $\mathbf{i(t)} \approx 1.49e^{j(3t - 63.43°)}$ A.

If the source is $V_{s1}(t) = 10\cos(3t) = \mathrm{Re}[10e^{j3t}$ V.$] = \mathrm{Re}[\mathbf{V_s(t)}]$,
the response is $i_1(t) \approx 1.49\cos(3t - 63.43°)$ A.
$= \mathrm{Re}[1.49e^{j(3t - 63.43°)}$ A.$] = \mathrm{Re}[\mathbf{i(t)}]$

If the source is $V_{s2}(t) = 10\sin(3t) = \mathrm{Im}[10e^{j3t}$ V.$] = \mathrm{Im}[\mathbf{V_s(t)}]$,
the response is $i_1(t) \approx 1.49\sin(3t - 63.43°)$ A.
$= \mathrm{Im}[1.49e^{j(3t - 63.43°)}] = \mathrm{Im}[\mathbf{i(t)}]$

Note that the interrelationship between all three sources and their current-response counterparts is *always* the same, namely Euler's Equation!

5.4 Use of Complex Exponential Source cont.

The last theoretical consideration to explore is why the exponential source case is worthy of consideration at all

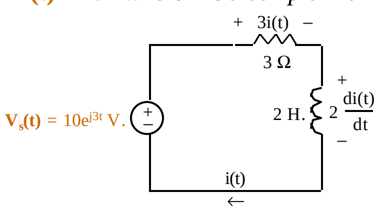
The reason is that the exponential source case provides a computationally compact means for obtaining particular solutions for sinusoidally driven circuits—in other words, it's computationally expedient!

To see this, consider the circuit analyzed earlier—but now driven instead by the exponential counterpart of its original sinusoidal source, namely ...

5.4 Use of Complex Exponential Source cont.

The differential equation is: $2(di(t)/dt) + 3i(t) = \mathbf{V_s(t)}$

Let the particular solution *resemble* the source so that it too is a complex exponential differing from the source only in magnitude and phase angle; i.e., let $\mathbf{i(t)} = \mathbf{I}e^{j3t}$ where **I** is a *complex number*



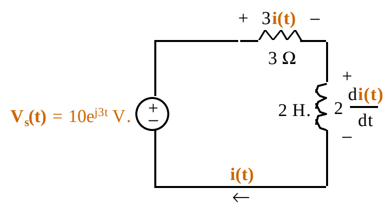
5.4 Use of Complex Exponential Source cont.

Then $2(d\mathbf{i(t)}/dt) + 3\mathbf{i(t)} = \mathbf{V_s(t)}$ or ...

$2(d(\mathbf{I}e^{j3t})/dt) + 3(\mathbf{I}e^{j3t}) = 10e^{j3t}$ V. or…

$2(j3)\mathbf{I}e^{j3t} + 3(\mathbf{I}e^{j3t}) = 10e^{j3t}$ V. or …

$(3 + j6)\mathbf{I}e^{j3t} = 10e^{j3t}$ V. and the e^{j3t} factors (time functions) cancel leaving …

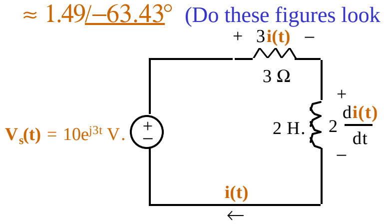


5.4 Use of Complex Exponential Source cont.

$\mathbf{I} = 10$ V./(3 + j6)—which is an equation in complex numbers *only* (no time functions at all) which yields the magnitude and phase angle unique to the current response, namely …

$\mathbf{I} = 10\angle 0°$ V./(3 + j6) ≈ (10∠0°)/(6.71∠63.43°)

≈ 1.49∠–63.43° (Do these figures look familiar?)



5.4 Use of Complex Exponential Source cont.

Therefore, $\mathbf{i(t)} = \mathbf{I}e^{j3t} \approx 1.49\angle{-63.43°}\,\mathbf{e^{j3t}}$ A.

$\approx 1.49e^{-j63.43°}\mathbf{e^{j3t}}$ A. $\approx 1.49\mathbf{e}^{j(3t-63.43°)}$ A.

and …

If $V_s(t) = 10\cos(3t)$ V. = Re[$10e^{j3t}$ V.], what's i(t)?
i(t) ≈ Re[$1.49\mathbf{e}^{j(3t-63.43°)}$] ≈ 1.49cos(3t – 63.43°) A.

Likewise

If $V_s(t) = 10\sin(3t)$ V. = Im[$10e^{j3t}$ V.], what's i(t)?
i(t) ≈ Im[$1.49\mathbf{e}^{j(3t-63.43°)}$] ≈ 1.49sin(3t – 63.43°) A.

The final task is to develop a method to streamline the calculations even further

This method is ...

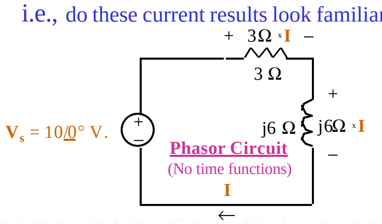
5.5 The Phasor Circuit

Recall that , $\mathbf{I} = 10$ V./(3 + j6) which is an equation in complex numbers (no time functions at all!)

How does this compare with the calculation of $\mathbf{I}$ = 10∠0° V./(3 + j6) Ω ≈ (10∠0° V.)/(6.71∠63.43° Ω)

≈ 1.49∠–63.43°A. for the *conceptual* circuit below;

i.e., do these current results look familiar?

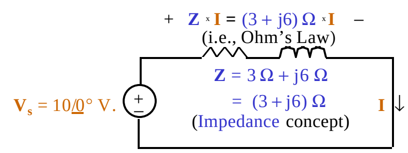


5.5 The Phasor Circuit cont.

The phasor circuit can be redrawn as shown below

Note that the phasor circuit provides a *(conceptual) schematic tool* for finding the magnitude and phase angle of the current (which are its only unique properties!)

Of the complex *numbers* $\mathbf{V_s}$, $\mathbf{Z}$ and $\mathbf{I}$, which do not correspond to time functions?



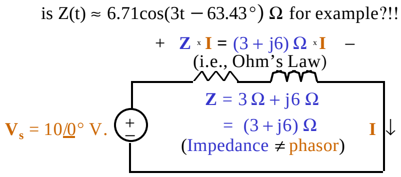
By KVL, $\mathbf{I} = \mathbf{V_s}/\mathbf{Z}$ = (10∠0° V.) / (3 + j6)Ω ≈ 1.49∠–63.43°A.

5.5 The Phasor Circuit cont.

Only currents and voltages are phasors
(i.e., have time-function counterparts)

Don't believe it? Then what's Z(t) for the phasor circuit below if the phasors correspond to cosinusoids
(so that $V_s(t) = 10\cos(3t)$ V. and i(t) ≈ 1.49cos(3t – 63.43°) A.)?

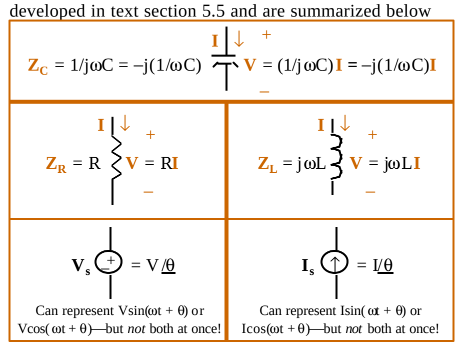
Since $\mathbf{Z}$ = (3 + j6) Ω ≈ 6.71∠63.43° Ω,
is Z(t) ≈ 6.71cos(3t – 63.43°) Ω for example?!!



By KVL, $\mathbf{I} = \mathbf{V_s}/\mathbf{Z}$ = (10∠0° V.) / (3 + j6)Ω ≈ 1.49∠–63.43°A.

5.5 The Phasor Circuit cont.

The "Rules and Regulations" for phasor sources and impedances are developed in text section 5.5 and are summarized below



5.5 The Phasor Circuit cont.

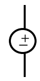
The following equations from text p.171 are useful in reconciling situations where both sinusoidal and cosinusoidal sources are present

$$\sin(\omega t + \theta) = \cos(\omega t + \theta - 90°)$$

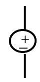
$$\cos(\omega t + \theta) = \sin(\omega t + \theta + 90°)$$

(see example below)

If $V\cos(\omega t + \theta)$'s phasor representation is ⇒



Then $E\sin(\omega t + \phi)$'s phasor representation is ⇒



5.5 The Phasor Circuit cont.—Example 5-1

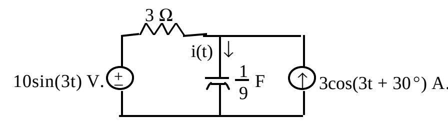
Find the AC steady-state current i(t)—see text problem 5.26 (p.204)

Let the phasors represent cosinusoids so that the current source's phasor current is $3\angle 30°$ A.

Then the voltage source's phasor voltage is $10\angle -90°$ V.

The capacitor's impedance is $-j\{1/[(3 \text{ Rad./sec.})(1/9 \text{ F})]\} = -j3\ \Omega$

So the phasor circuit becomes ...



5.5 The Phasor Circuit cont.—Example 5-1 cont.

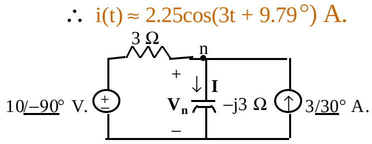
KCL at node n:

$$(10\angle -90° \text{ V.} - \mathbf{V_n}) / 3\Omega - \mathbf{V_n} / (-j3\ \Omega) + 3\angle 30° \text{ A.} = 0 \text{ A.}$$

Which has solution $\mathbf{V_n} \approx 6.745\angle -80.21°$ V.

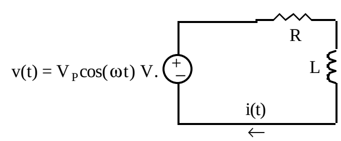
Then $\mathbf{I} = \mathbf{V_n} / (-j3\ \Omega) \approx 2.25\angle +9.79°$ A.

∴ $i(t) \approx 2.25\cos(3t + 9.79°)$ A.



5.6 Average Power, Reactive Power and Power Factor

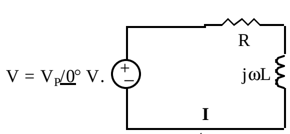
What's the nature of power in AC steady-state circuits? Consider the following RL circuit … What's the circuit's *instantaneous* power P(t)?



5.6 Average Power, Reactive Power and Power Factor cont.

Applying KVL to the phasor circuit below yields the current's phasor from which i(t) is deduced

Then the *instantaneous* power is P(t) = v(t)i(t)



$$\mathbf{I} = \frac{V_P\angle 0°}{R + j\omega L} = \frac{V_P\angle 0°}{Z\angle\theta} = \frac{V_P}{Z}\angle -\theta = I_P\angle -\theta \quad \therefore\ i(t) = I_p\cos(\omega t - \theta) \text{ A.}$$

Where $Z = [R^2 + (\omega L)^2]^{½}$ and $\theta = \tan^{-1}(\omega L/R)$

5.6 Average Power, Reactive Power and Power Factor cont.

$$P(t) = v(t)i(t) = V_P\cos(\omega t)I_P\cos(\omega t - \theta)$$
$$= 0.5V_PI_P[\cos(\theta) + \cos(2\omega t - \theta)]$$
$$= 0.5V_PI_P[\cos(\theta) + \cos(2\omega t)\cos(\theta) - \sin(2\omega t)\sin(\theta)]$$
$$= 0.5V_PI_P\cos(\theta)\{1 + \cos(2\omega t)\} + 0.5V_PI_P\sin(\theta)\{-\sin(2\omega t)\}$$

Recall from text p. 39 that $X_{RMS} = X_{PEAK}/\sqrt{2}$ ∴ since $2 = \sqrt{2}\sqrt{2}$

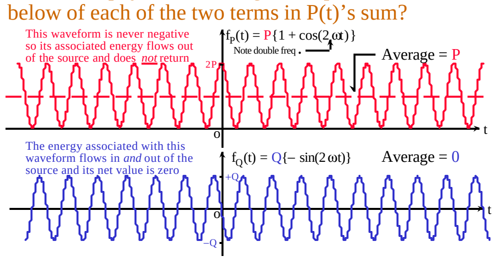
$$P(t) = V_{RMS}I_{RMS}\cos(\theta)\{1 + \cos(2\omega t)\} + V_{RMS}I_{RMS}\sin(\theta)\{-\sin(2\omega t)\}$$

∴ $P(t) = Pf_P(t) + Qf_Q(t)$ Where …

$P = V_{RMS}I_{RMS}\cos(\theta)$ and $Q = V_{RMS}I_{RMS}\sin(\theta)$ are *constants* and $f_P(t) = \{1 + \cos(2\omega t)\}$ and $f_Q(t) = -\sin(2\omega t)$ are *time functions*

5.6 Average Power, Reactive Power and Power Factor cont.

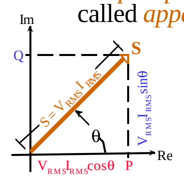
What's the physical meaning of the plots shown below of each of the two terms in P(t)'s sum?



N.B.: The average of $P(t) = Pf_P(t) + Qf_Q(t)$ is P ∴ P is *average power*

5.6 Average Power, Reactive Power and Power Factor cont.

P and Q as sides of the *Power Triangle and*
P and Q as the real and imaginary parts of *complex power* **S** (whose magnitude S is called *apparent power*)



$\mathbf{S} = P + jQ = V_{RMS}I_{RMS}\cos\theta + jV_{RMS}I_{RMS}\sin\theta$

$\mathbf{S} = Se^{j\theta} = S\underline{/\theta} = V_{RMS}I_{RMS}\underline{/\theta}$

N.B.: θ is called the *power factor angle* where the power factor is either cos(θ) lag if $\theta > 0$, cos(θ) lead if $\theta < 0$ or unity ($\theta = 0$)

5.6 Average Power, Reactive Power and Power Factor cont.

What's *complex power* **S** in terms of RMS voltage and current phasors

What are RMS phasors?

An RMS phasor is it's traditional (peak) phasor counterpart divided by √2 (see text p. 39)

Then *complex power* $\mathbf{S} = \mathbf{V}_{RMS}\mathbf{I}_{RMS}{}^*$

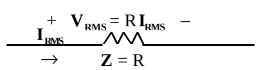
Where $\mathbf{V}_{RMS} = \mathbf{V}/\sqrt{2}$ and $\mathbf{I}_{RMS} = \mathbf{I}/\sqrt{2}$

5.6 Average Power, Reactive Power and Power Factor cont.

Then since *complex power* $\mathbf{S} = \mathbf{V}_{RMS}\mathbf{I}_{RMS}{}^*$ be sure to

Remember: Before making any power related calculations from phasors *always* convert traditional (peak) phasors to RMS phasors!!

5.6 Average Power, Reactive Power and Power Factor— some useful formulae



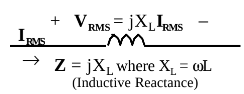
$\mathbf{S}_R = \mathbf{V}_{RMS}\mathbf{I}_{RMS}{}^*$

$= (R\mathbf{I}_{RMS})\mathbf{I}_{RMS}{}^* = \mathbf{V}_{RMS}(\mathbf{V}_{RMS}/R)^*$

$= RI^2_{RMS} = V^2_{RMS}/R = P_R + j0$

∴ $\mathrm{Re}(\mathbf{S}_R) = P_R \geq 0$ and $\mathrm{Im}(\mathbf{S}_R) = Q_R = 0$ for resistors

5.6 Average Power, Reactive Power and Power Factor— some useful formulae cont.



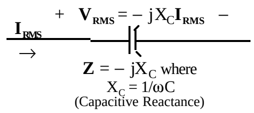
$\mathbf{S}_L = \mathbf{V}_{RMS}\mathbf{I}_{RMS}{}^*$

$= (jX_L\mathbf{I}_{RMS})\mathbf{I}_{RMS}{}^* = \mathbf{V}_{RMS}[\mathbf{V}_{RMS}/(jX_L)]^*$

$= jX_LI^2_{RMS} = jV^2_{RMS}/X_L = 0 + jQ_L$

∴ $\mathrm{Re}(\mathbf{S}_L) = P_L = 0$ and $\mathrm{Im}(\mathbf{S}_L) = Q_L \geq 0$ for inductors

5.6 Average Power, Reactive Power and Power Factor— some useful formulae cont.



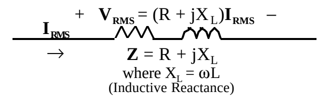
$S_C = V_{RMS} I_{RMS}^*$

$= (-jX_C I_{RMS}) I_{RMS}^* = V_{RMS}(V_{RMS}/(-jX_C))^*$

$= -jX_C I^2_{RMS} = -jV^2_{RMS}/X_C = 0 + jQ_C$

$\therefore$ Re(S_C) = $P_C = 0$ and Im(S_C) = $Q_C \leq 0$ for capacitors

5.6 Average Power, Reactive Power and Power Factor— some useful formulae cont.



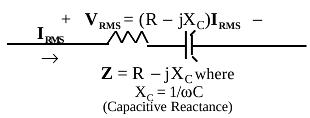
$S = V_{RMS} I_{RMS}^*$

$= [(R + jX_L)I_{RMS}]I_{RMS}^* = (R + jX_L)I^2_{RMS}$

$= RI^2_{RMS} + jX_L I^2_{RMS} \quad = P + jQ_L$

$\therefore P = RI^2_{RMS}$ and $Q_L = X_L I^2_{RMS}$ for this scenario

5.6 Average Power, Reactive Power and Power Factor— some useful formulae cont.



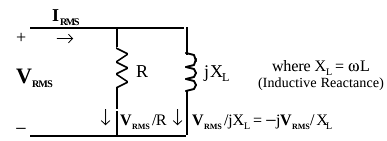
$S = V_{RMS} I_{RMS}^*$

$= [(R - jX_C)I_{RMS}]I_{RMS}^* = (R - jX_C)I^2_{RMS}$

$= RI^2_{RMS} + j(-X_C I^2_{RMS}) = P + jQ_C$

$\therefore P = RI^2_{RMS}$ and $Q_C = -X_C I^2_{RMS}$ for this scenario

5.6 Average Power, Reactive Power and Power Factor— some useful formulae cont.



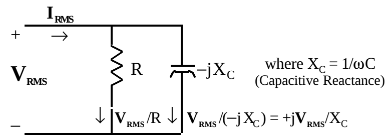
$S = V_{RMS} I_{RMS}^*$

$= V_{RMS}(V_{RMS}/R - jV_{RMS}/X_L)^* = V_{RMS}(V_{RMS}^*/R + jV_{RMS}^*/X_L)$

$= V^2_{RMS}/R + jV^2_{RMS}/X_L = P + jQ_L$

$\therefore P = V^2_{RMS}/R$ and $Q_L = V^2_{RMS}/X_L$ for this scenario

5.6 Average Power, Reactive Power and Power Factor— some useful formulae cont.



$S = V_{RMS} I_{RMS}^*$

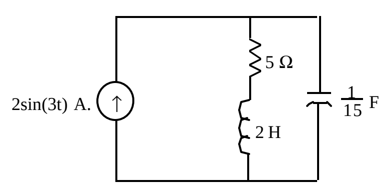
$= V_{RMS}(V_{RMS}/R + jV_{RMS}/X_C)^* = V_{RMS}(V_{RMS}^*/R - jV_{RMS}^*/X_C)$

$= V^2_{RMS}/R + j(-V^2_{RMS}/X_C) = P + jQ_C$

$\therefore P = V^2_{RMS}/R$ and $Q_C = -V^2_{RMS}/X_L$ for this scenario

5.6 Average Power, Reactive Power and Power Factor—Example 5-2

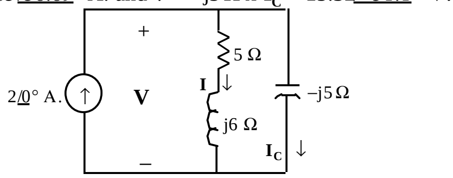
Calculate the complex power produced by the source and consumed by each passive element (see text problem 5.31, p. 205)



5.6 Average Power, Reactive Power and Power Factor—Example 5-2 cont.

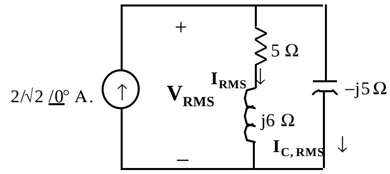
Phasor circuit: ω = 3 Rad./Sec., $j\omega L = j3\times2 = j6\ \Omega$, $-j(1/\omega C) = -j5\ \Omega$, and the source phasor is $2\underline{/0°}$ A.

By current ÷, $\mathbf{I} = [-j5\ \Omega/(5\ \Omega + j6\ \Omega - j5\ \Omega)]2\underline{/0°}$ A. Ergo, $\mathbf{I} \approx 1.961\underline{/-101.3°}$ A. and by KCL, $\mathbf{I}_C = 2\underline{/0°}\ \text{A.} - \mathbf{I}$ Ergo $\mathbf{I}_C \approx 3.063\underline{/38.89°}$ A. and $\mathbf{V} = -j5\ \Omega \times \mathbf{I}_C \approx 15.32\underline{/-51.1°}$ V.



5.6 Average Power, Reactive Power and Power Factor—Example 5-2 cont.

Dividing each current and voltage phasor by √2 yields the RMS phasors: $1.414\underline{/0°}$ A. (source phasor), $I_{RMS} \approx 1.387\underline{/-101.3°}$ A., $I_{C,RMS} \approx 2.166\underline{/38.89°}$ A. and $V_{RMS} \approx 10.83\underline{/-51.1°}$ V.

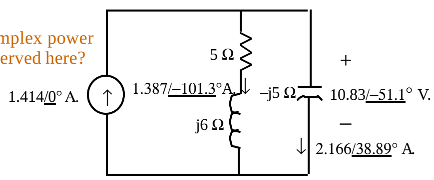


5.6 Average Power, Reactive Power and Power Factor—Example 5-2 cont.

Then the source's complex power is $\mathbf{V}_{RMS}\ (1.414\underline{/0°}\ \text{A.})^*$ $\approx 15.31\underline{/-51.1°}$ VA., ≈ 9.61W − j11.91 VAR

The complex power of the resistor, inductor and capacitor are respectively $(I_{RMS})^2R = (1.387\ \text{A.})^2(5\Omega) \approx 9.61\text{W}$, $j(I_{RMS})^2X_L = j(1.387\ \text{A.})^2(6\Omega) \approx j11.52$ VAR and $-j(V_{RMS})^2/X_C = -j(10.83\ \text{V.})^2/5\Omega \approx -j23.46$ VAR

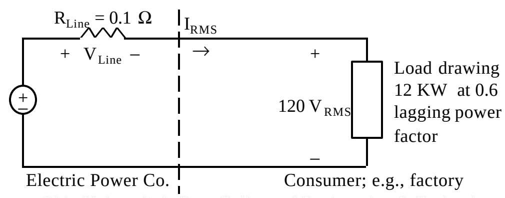
Is complex power conserved here?



5.6 Average Power, Reactive Power and Power Factor Correction: An Application

$S = P/\cos(\theta) = 12\ \text{KW}/0.6 = 20\ \text{KVA} = V_{RMS}I_{RMS}$
$\therefore I_{RMS} = 20\ \text{KVA}/120\text{V} \approx 167$ A. (RMS)

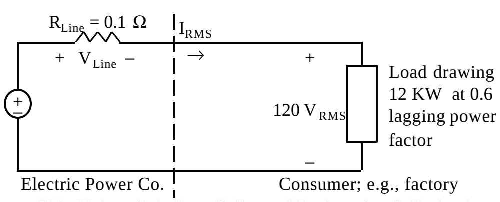
Ergo, Line losses = $I^2_{RMS}R_{Line} \approx 2.78$ KW (≈ 23% of load power)
and $V_{Line} = R_{Line}I_{RMS} \approx 16.7$ V (≈ 14% of load voltage)



5.6 Average Power, Reactive Power and Power Factor Correction: An Application cont.

The load's complex power is $\mathbf{S}_{Load} = P + jQ$
$= 12\ \text{KW} + j\,|\mathbf{S}_{Load}|\sin(\theta) = 12\ \text{KW} + j20\ \text{KVA} \times 0.8$
$= 12\ \text{KW} + j16\ \text{KVAR}$

Next, add -j7 KVAR of capacitance in parallel with the load to obtain ...

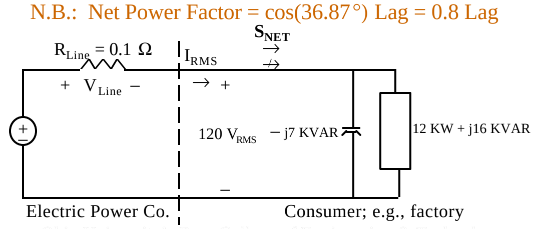


5.6 Average Power, Reactive Power and Power Factor Correction: An Application cont.

The net (load and capacitor) complex power is:

$\mathbf{S}_{NET} = \mathbf{S}_{Load} - j7\ \text{KVAR} = 12\ \text{KW} + j16\text{KVAR} - j7\ \text{KVAR}$
$= 12\ \text{KW} + j9\ \text{KVAR} \approx 15\underline{/36.87°}$ KVA

N.B.: Net Power Factor = cos(36.87°) Lag = 0.8 Lag

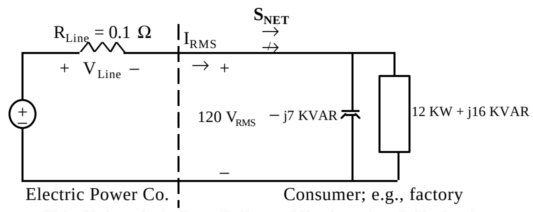


5.6 Average Power, Reactive Power and Power Factor Correction: An Application cont.

Then S_{NET} = 15 KVA = V_{RMS} I_{RMS} = (120 V_{RMS}) I_{RMS} yields I_{RMS} = 15 KVA/120 V. = 125 A. (RMS) Ergo, ...

Line losses = $I^2_{RMS} R_{Line} \approx$ 1.56 KW (≈ 13% of load power) and $V_{Line} = R_{Line} I_{RMS} \approx$ 12.5 V (≈ 10.4% of load voltage)

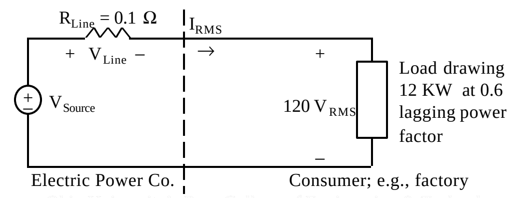
Who saves how much (annually) if electricity costs 10¢ per KWh?



5.6 Average Power, Reactive Power and Power Factor Correction: An Application

Example 5-3

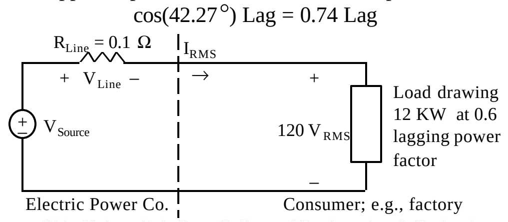
For the previous power factor correction application, what's the source's apparent power, power factor and voltage magnitude with and without the power factor correction?



5.6 Average Power, Reactive Power and Power Factor Correction: An Application

Example 5-3 cont.

Without correction, the source produces $\mathbf{S}_{Source}$ = $\mathbf{S}_{Load}$ + $\mathbf{S}_{Line}$ = (12 KW + j16 KVAR) + (2.78 KW + j0 KVAR) = 14.78 KW +j16 KVAR = 21.78 /42.27° KVA ∴ the apparent power is 21.78 KVA and the power factor is cos(42.27°) Lag = 0.74 Lag

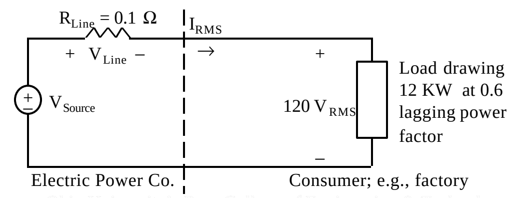


5.6 Average Power, Reactive Power and Power Factor Correction: An Application

Example 5-3 cont.

The current is I_{RMS} = 167 A. and the apparent power ($V_{Source} \times I_{RMS}$) is 21.78 KVA

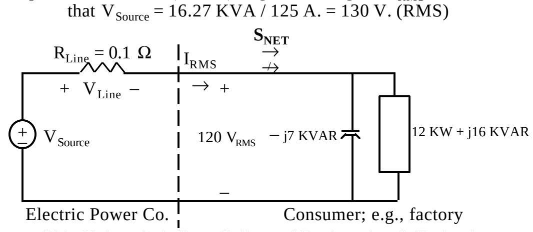
∴ V_{Source} = 21.78 KVA / 167 A. = 130 V. (RMS)



5.6 Average Power, Reactive Power and Power Factor Correction: An Application

Example 5-3 cont.

With correction, the source produces $\mathbf{S}_{Source}$ = $\mathbf{S}_{Net}$ + $\mathbf{S}_{Line}$ = (12 KW + j9 KVAR) + (1.56 KW + j0 KVAR) = 13.56 KW + j9 KVAR = 16.27/33.57° KVA ∴ the apparent power is 16.27 KVA, the power factor is cos(33.57°) Lag = 0.83 Lag and I_{RMS} = 125 A. so that V_{Source} = 16.27 KVA / 125 A. = 130 V. (RMS)



5.6 Average Power, Reactive Power and Power Factor Correction: An Application

Example 5-3 cont.

Summary

PF Correction	Apparent Power[1]	Power Factor[2]	Source Voltage
Without	21.78 KVA	0.74 Lag	130 V.
With	16.27 KVA	0.83 Lag	130 V.

[1] Apparent power is germane to the source's *size* (rating) and therefore its capital cost

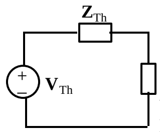
[2] The power factor indicates the percentage of the source's *apparent* power that is *real* power (83% versus 74%) and thus is a measure of the source's efficacy

5.6 Average Power, Reactive Power and Power Factor—Maximum Power Transfer

The steady-state AC version of the maximum power transfer theorem is similar to its DC counterpart (but the numbers are complex) and is shown below—see text pp. 183-185 for its (calculus) derivation

Note that both the (Thevenin) phasor circuit's voltage $\mathbf{V}_{Th}$ and impedance $\mathbf{Z}_{Th}$ are complex numbers

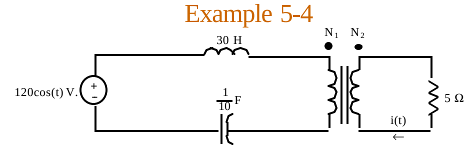
The rules for obtaining the Thevenin circuit remain: $\mathbf{V}_{Th} = \mathbf{V}_{open\ circuit}$ and $\mathbf{Z}_{Th}$ = circuit impedance remaining after all independent sources are deactivated



$\mathbf{Z}_{Load} = \mathbf{Z}^*_{Th}$ for maximum (average) power transfer from the Thevenin circuit to the load— $R_{Th} = Z_{Th}$ (magnitude of $\mathbf{Z}_{Th}$) if the load is purely resistive

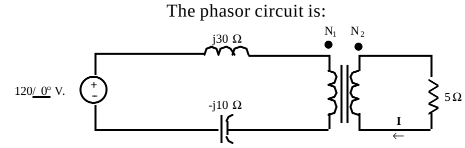
5.6 Average Power, Reactive Power and Power Factor—Maximum Power Transfer cont.

Example 5-4



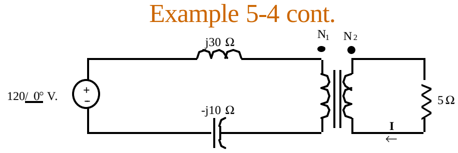
Calculate the turns ratio N_1/N_2 which will maximize the average power consumed by the resistor, the corresponding maximum average power and the resistor current i(t)

The phasor circuit is:

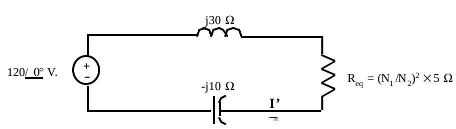


5.6 Average Power, Reactive Power and Power Factor—Maximum Power Transfer cont.

Example 5-4 cont.

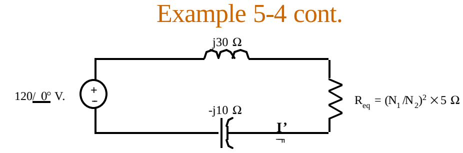


Reflecting the resistor across the transformer yields:

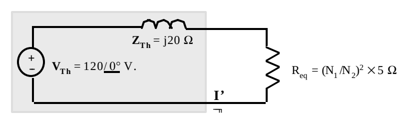


5.6 Average Power, Reactive Power and Power Factor—Maximum Power Transfer cont.

Example 5-4 cont.

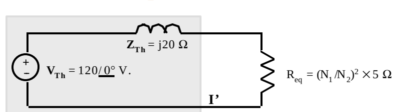


Combining the two series reactances yields the (resistance-loaded) Thevenin circuit:



5.6 Average Power, Reactive Power and Power Factor—Maximum Power Transfer cont.

Example 5-4 cont.



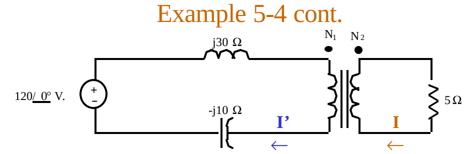
For maximum average power, $R_{eq} = (N_1/N_2)^2 \times 5\ \Omega = |\mathbf{Z}_{Th}| = 20\ \Omega$

$\therefore N_1/N_2 = 2$

Then $\mathbf{I}' = [\mathbf{V}_{Th} = 120\angle 0°\ \text{V.}] / [\mathbf{Z}_{Th} + R_{eq} = (20 + j20)\ \Omega] = 3\sqrt{2}\angle -45°$ A.

5.6 Average Power, Reactive Power and Power Factor—Maximum Power Transfer cont.

Example 5-4 cont.



As a result of the transformer's turns ratio, $\mathbf{I} = (N_1/N_2)\ \mathbf{I}'$

$\therefore \mathbf{I} = (N_1/N_2)\ \mathbf{I}' = 2 \times 3\sqrt{2}\angle -45°$ A. $= (6\sqrt{2})\angle -45°$ A. so that

$i(t) = (6\sqrt{2})\cos(t - 45°)$ A. » $8.485\cos(t - 45°)$ A.

and $I_{RMS} = |\mathbf{I}|/\sqrt{2} = 6$ A. so that $P_{ave.,\,max.} = (I_{RMS})^2 \times 5\ \Omega$ yields:

$P_{ave.,\,max.} = 180$ W.

5.7 Frequency Response of Circuits

Time-domain signals have a frequency-domain counterpart; e.g., music is a time varying sound signal with, inter alia, bass, mid-range and treble characteristics (components)

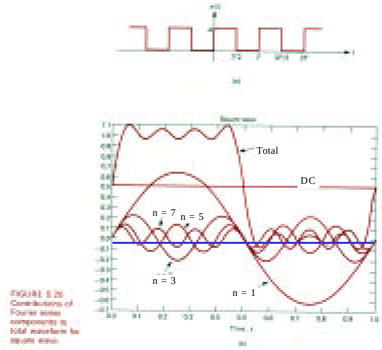
The concomitant mathematical relationships were discovered in 1822 by Jean Baptiste Joseph Fourier (1768-1830)

See text equation 5.129 (p. 185) and its accompanying Figure 5.20 (p. 186) = next slide, as well as text Appendix C (p. 753)

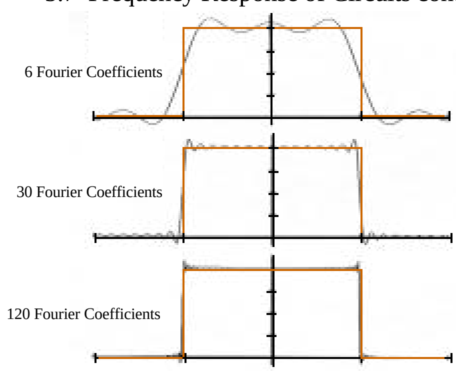
Signals of all sorts (e.g., sound) can be converted to electrical signals (voltage and current waveforms) via transducers

Electrical signals can be "processed" by electric circuits because electric circuit behavior is frequency dependent

5.7 Frequency Response of Circuits cont.



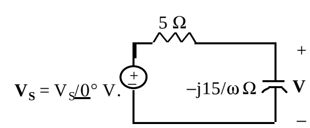
5.7 Frequency Response of Circuits cont.



5.7 Frequency Response of Circuits cont.

Consider the circuit of text Figure P5.39 (p. 207) whose phasor circuit is shown below

Voltage division produces the ratio (*transfer function*) $\mathbf{H}(j\omega) = \mathbf{V}/\mathbf{V}_S = (-j15/\omega)/(5 - j15/\omega) = 1/(1 + j\omega/3)$ which in this case is *gain* (i.e., what are **H**'s units?)



5.7 Frequency Response of Circuits cont.

This *transfer function* can be written in polar form as $\mathbf{H}(j\omega) = M(\omega)\underline{/\theta(\omega)}$ where:

$M(\omega) = \{1/[1 + (\omega/3)^2]^{½}\}$ and $\theta(\omega) = -\tan^{-1}(\omega/3)$

So both the gain's magnitude and phase angle depend on frequency; i.e., $M(0) = 1$ (which is also the M's maximum) and $\theta(0) = 0°$ whereas $\text{Lim}[M(\omega)]$ and $\text{Lim}[\theta(\omega)]$ as $\omega \rightarrow \infty$ are 0 and $-90°$ respectively

Also, for $(\omega/3) >> 1$, $M(\omega) \approx 3/\omega$

What's $M(\omega)$ and $\theta(\omega)$ when $\omega = 3$ Rad./Sec.?

5.7 Frequency Response of Circuits cont.

$M(3) = \{1/[1 + (3/3)^2]^{½}\} = 1/\surd 2$ and $\theta(\omega) = -45°$ which is the transfer function's *half-power point*

Why the moniker *half-power point*?

Suppose $\mathbf{V}_S = V_S\underline{/0°}$ V. for all frequencies (i.e., all ω)

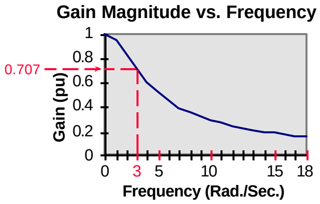
Then the maximum output voltage $V_{max.} = V_S$ occurs at $\omega = 0$ since $V = M(\omega)V_S$ and $M_{max.} = M(0) = 1$

Impressing this voltage across a load resistor yields the maximum output power $P_{max.} = V_S{}^2/R_L$

The output voltage at $\omega = 3$ Rad./Sec. is $M(3)V_S = V_S/\surd 2$ and the corresponding power is $(V_S/\surd 2)^2/R_L = [V_S{}^2/R_L]/2 = P_{max.}/2$ (*half of $P_{max.}$*)

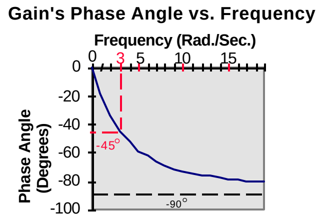
5.7 Frequency Response of Circuits cont.

A plot of $M(\omega)$ is shown below



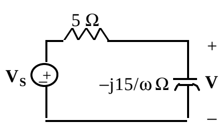
5.7 Frequency Response of Circuits cont.

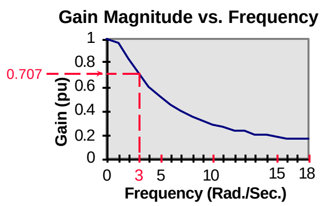
A plot of $\theta(\omega)$ is shown below



5.7.2 Filters

The RC circuit just considered is a *low-pass filter* because the magnitude of its gain $M(\omega)$ *favors lower frequencies*





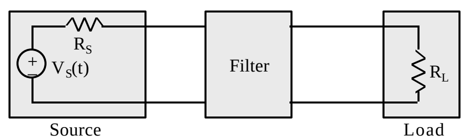
Transfer function's gain magnitude = $|\mathbf{H}(j\omega)| = |\mathbf{V}/\mathbf{V}_S| = V/V_S = M(\omega)$

N.B.: If the capacitor's voltage favors low frequencies, one can intuit that the resistor's voltage favors high frequencies (why?)

5.7.2 Filters cont.

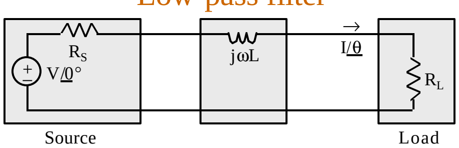
Other types of filters are possible and are developed in text section 5.7.2 (pp. 190-200) using the generic prototype shown below

The *results* for each filter are ...



5.7.2 Filters cont.

Low-pass filter



$\mathbf{H}(j\omega) = I/\theta \,/\, V/0° = \{V/0° \,/\, (R_S + R_L + j\omega L)\} / V/0°$

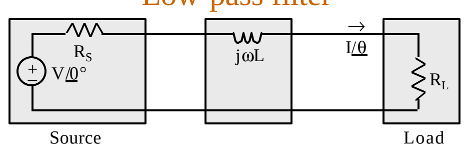
for which

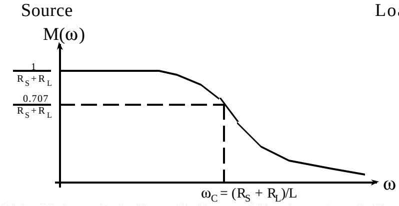
$$M(\omega) = \frac{1}{R_S + R_L} \cdot \frac{1}{(1 + [\omega L/(R_S + R_L)]^2)^{1/2}}$$

Which has a cut-off (half-power) frequency of $\omega_C = (R_S + R_L)/L$

5.7.2 Filters cont.

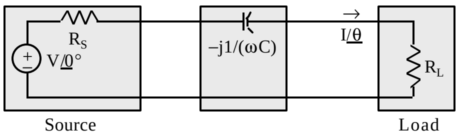
Low-pass filter





5.7.2 Filters cont.

High-pass filter



$\mathbf{H}(j\omega) = I\underline{/\theta} / V\underline{/0^\circ} = \{ V\underline{/0^\circ} / [R_S + R_L - j(1/\omega C)]\} / V\underline{/0^\circ}$

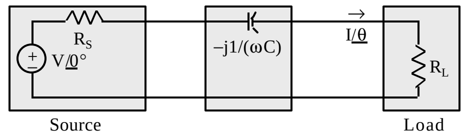
for which

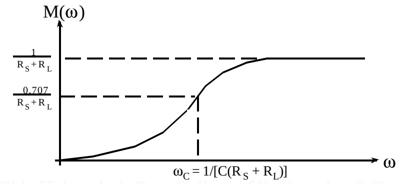
$$M(\omega) = \frac{1}{R_S + R_L} \cdot \frac{1}{\{1 + [1/(\omega C(R_S + R_L))]^2\}^{1/2}}$$

Which has a cut-off (half-power) frequency of $\omega_C = 1/[C(R_S + R_L)]$

5.7.2 Filters cont.

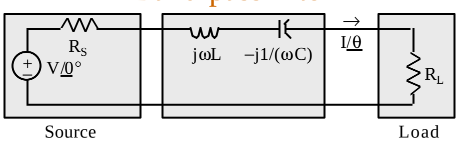
High-pass filter





5.7.2 Filters cont.

Band-pass filter



$\mathbf{H}(j\omega) = I\underline{/\theta} / V\underline{/0^\circ} = \{ V\underline{/0^\circ} / [R_S + R_L + j\omega L - j(1/\omega C)]\} / V\underline{/0^\circ}$

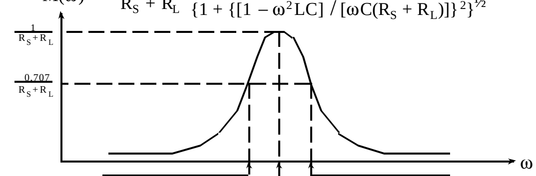
for which

$$M(\omega) = \frac{1}{R_S + R_L} \cdot \frac{1}{\{1 + \{[1 - \omega^2 LC] / [\omega C(R_S + R_L)]\}^2\}^{1/2}}$$

5.7.2 Filters cont.

Band-pass filter

$$M(\omega) = \frac{1}{R_S + R_L} \cdot \frac{1}{\{1 + \{[1 - \omega^2 LC] / [\omega C(R_S + R_L)]\}^2\}^{1/2}}$$



$\omega_l = -\frac{R_S + R_L}{2L} + \left[\left(\frac{R_S + R_L}{2L}\right)^2\right]^{1/2}$ (Lower half-power frequency)

$\omega_0 = 1/(LC)^{1/2}$ (Center frequency)

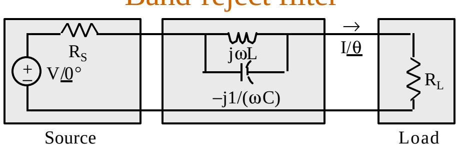
$\omega_u = +\frac{R_S + R_L}{2L} + \left[\left(\frac{R_S + R_L}{2L}\right)^2\right]^{1/2}$ (Upper half-power frequency)

$B = \omega_u - \omega_l = (R_S + R_L)/L$ (Bandwidth)

$Q = \omega_0 / B = \omega_0 L/(R_S + R_L)$ (Selectivity)

5.7.2 Filters cont.

Band-reject filter



$\mathbf{H}(j\omega) = I\underline{/\theta} / V\underline{/0^\circ} = V\underline{/0^\circ} / [R_S + R_L + (j\omega L) \,||\, (-j(1/\omega C))]$

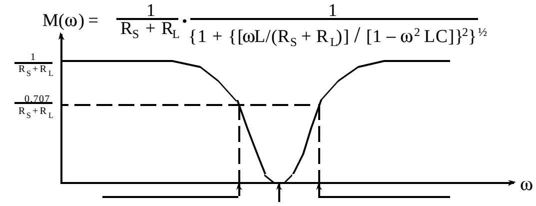
for which

$$M(\omega) = \frac{1}{R_S + R_L} \cdot \frac{1}{\{1 + \{[\omega L/(R_S + R_L)] / [1 - \omega^2 LC]\}^2\}^{1/2}}$$

5.7.2 Filters cont.

Band-reject filter

$$M(\omega) = \frac{1}{R_S + R_L} \cdot \frac{1}{\{1 + \{[\omega L/(R_S + R_L)] / [1 - \omega^2 LC]\}^2\}^{1/2}}$$



$\omega_l = -\alpha + (\alpha^2 + \omega_0^2)^{1/2}$ (Lower half-power frequency)

$\omega_0 = 1/(LC)^{1/2}$ (Center frequency)

$\omega_u = \alpha + (\alpha^2 + \omega_0^2)^{1/2}$ (Lower half-power frequency)

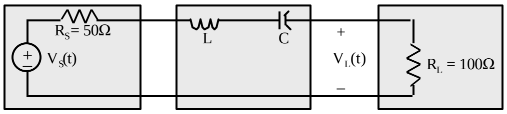
$\alpha = \frac{1}{2C(R_S + R_L)}$

$B = \omega_u - \omega_l = 1/[C(R_S + R_L)]$ (Bandwidth)

$Q = \omega_0 / B = \omega_0 C(R_S + R_L)$ (Selectivity)

5.7.2 Filters cont.

Example 5-5



Design the band-pass filter to have a center frequency of 1 MHz. and a bandwidth of 10 kHz. What's this filter's Q?

$\omega_0 = 2\pi f_0 = 2\pi\times10^6$ Rad./Sec. $= 1/(LC)^{½}$ = (Center frequency)

$B = 2\pi f_{BW} = 2\pi\times10^4$ Rad./Sec. $= (R_S + R_L)/L = 150\,\Omega/L$ (Bandwidth)

$\therefore\ L = 150\,\Omega / (2\pi\times10^4$ Rad./Sec.$) \approx 2.39$ mH

and $C = 1 / (\omega_0^2 L) \approx 1/[(2\pi\times10^6$ Rad./Sec.$)^2\times2.39$ mH$] \approx 10.6$ pF

and $Q = \omega_0 / B = (2\pi\times10^6$ Rad./Sec.$) / (2\pi\times10^4$ Rad./Sec.$) = 100$

We welcome your questions with Enthusiasm!!

School of Electrical Engineering and Computer Science

Ohio University