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Wrench-Based Analysis of Cable-Driven Robots

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Abstract— **This paper introduces the available net wrench set, which is the set of all wrenches that a cable robot can apply to its surroundings without violating tension limits in the cables. This set is used as a framework for understanding and computing wrench-feasibility of a pose of the robot, allowing wrench-feasibility to be tested in the task space. The geometric properties of the available net wrench set are then exploited to permit simple geometric calculation of the boundaries of the wrench-feasible workspace. This workspace generation is extended to two other workspaces as well. Other design tools, including payload specification and failure analysis, are also presented.**

I. INTRODUCTION

Cable-driven robots, referred to as cable robots in this paper, are a type of robotic manipulator that has recently attracted interest for large workspace manipulation tasks. Cable robots are relatively simple in form, with multiple cables attached to a mobile platform or end-effector as illustrated in Figure 1. The end-effector is manipulated by motors that can extend or retract the cables. These motors may be in fixed locations or mounted to mobile bases. The end-effector may be equipped with various attachments, including hooks, cameras, electromagnets and robotic grippers. Figure 1 illustrates a cable robot with four cables equipped with a robotic gripping tool grasping a barrel.

Cable robots possess a number of desirable attributes which make them well-suited for a variety of applications. Because the motors may reel out a large amount of cable they can have very large workspaces. Because the motors do not need to be mounted near the end-effector they are suitable for operating in hazardous environments. Their load capacity can be very high, in some cases comparable to construction cranes. Their high payload-to-weight ratio make them attractive for high-speed manipulation tasks. Their simple design make them inexpensive, modular, transportable and easily reconfigurable.

Despite these characteristics, there have been relatively few cable robots used in practical applications. Three examples of cable robots that are currently in use are the Skycam [1], Intelligent Spreader Bar [2] and the NIST RoboCrane [3]. The Skycam is a broadcast-quality robotic camera suspended from a cable-driven computerized transport system with joystick control and is used in stadiums and indoor arenas. The Intelligent Spreader Bar is a six-cable spatial cable robot designed for transferring cargo at sea from one ship to another The NIST RoboCrane is a large-workspace robot for painting and maintaining aircraft which is also suitable for material handling in warehouses and storage facilities.

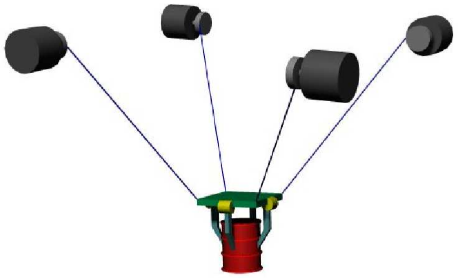

Fig. 1. Example Cable Robot.

Given the potential for cable robots to be used in a variety of applications, it is important to have tools for analyzing such mechanisms. Previous researchers have defined a variety of workspaces for cable robots in general analytical terms but there have been few efforts to develop effective methods for calculating these workspaces. In this paper, a wrench-based analysis approach is presented in Section III. Using this analysis, simple geometric methods can be applied to evaluate a given pose and generate the wrench-feasible workspace (Section IV). In addition, other workspaces can be formed using this technique (Section V) and several design tools are developed from this analysis (Section VI). These results are discussed (Section VII) and conclusions and future work are presented (Section VIII).

II. RELATED WORK

There have been several researchers who have developed analytical tools for cable robots that relate to the wrench-based analysis presented here. Shen et al. [4] examined a 3-cable planar cable robot with point-mass end-effector and formed a "set of manipulating forces," the set of forces that the 3 cables could exert on the end-effector. That set is a special case of the wrench set defined in this paper. Barrette and Gosselin [5] defined a similar set of wrenches they described as a "pseudo-pyramid." This pseudo-pyramid includes the set of all wrenches that the cables could apply to the end-effector at a pose if the cables have no upper tension limits. This pseudo-pyramid was then used to define and calculate the "dynamic workspace" of the manipulator. This workspace is the set of all poses of the end-effector where the end-effector can be given a specific acceleration.

The Wrench-Feasible Workspace is defined by Ebert-Uphoff and Voglewede as the set of poses where the manipulator can

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counteract a specified set of wrenches [6]. Alp and Agrawal define the "statically reachable workspace" as the set of all end-effector poses that can be reached statically [7], which is a special case of the Wrench-Feasible Workspace. Verhoeven and Hiller define the "controllable workspace" as the set of all end-effector poses where the manipulator can exert a single specific wrench [8], which is also a special case of the Wrench-Feasible Workspace. Other workspace issues for cable robots have been addressed in [9] and [10]. Although many of the results for general cable robots apply to point-mass cable robots, there has been relatively little research specifically on point-mass cable robots. Gorman et al. [11] developed a point-mass cable robot that can be used with three or four cables, but their research has focused primarily on sliding-mode control of the manipulator.

Some researchers have also pointed out the similarity of cable robots to parallel robots and multi-fingered grasps ([12], [13], [6]). However, because of the differences that do exist, many analysis tools from parallel robots and grasping do not apply to cable robots. Regardless, this paper will attempt to remain consistent with the terminology established for parallel robots and grasping.

In addition, a companion paper [14] to this paper by Riechel and Ebert-Uphoff applies the approach presented in this paper to planar and spatial cable robots with point-mass end-effector and provides numerical results for the wrench-feasible workspace.

III. WRENCH SET ANALYSIS

In order to use a cable robot to accomplish desired tasks, the cables driving the end-effector must exert wrenches (force/moment combinations) on the end-effector. Based on the considered pose of the robot, it is possible to determine the set of all possible wrenches that the cables can apply to the end-effector and thus the set of all wrenches that the end-effector can apply to its surroundings. Section III-A will define this set of wrenches and show how a graphical construction of this set allows geometric analysis of the cable robot pose. Sections IV and V will show how this geometric analysis can be used to construct the workspaces described in Section II.

Note that in the analysis presented here it is assumed that the cables have negligible mass and do not stretch or sag, the end-effector is a single rigid body with known cable attachment points on the end-effector relative to the center of gravity, the locations of the attachments of the cables to the motors are known and each motor controls exactly one cable. Cable lengths, the direction of gravity and the resulting pose of the mechanism are also assumed to be known.

A. Available Net Wrench Set

Assuming positive tension in all cables, the Jacobian relationship for parallel robots holds for cable robots. Thus the set of wrenches that can be applied to the end-effector can be formed by examining the positive range-space of the transpose of the Jacobian matrix. This matrix describes the linear relationship between the cable tensions, $\vec{t} = \{t_1, \ldots, t_p\}^T$, and the resulting wrench, $\$^w = \left\{\vec{F}_{ee}^T \;\; \vec{\tau}_{ee}^T\right\}^T$, at the end-effector. The *cable wrench set*, CW, is defined as:

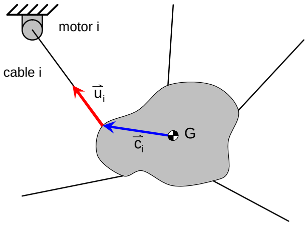


Fig. 2. Diagram of kinematic parameters.

$$CW = \left\{\$^w \, : \, \$^w = \boldsymbol{J}^T \vec{t}; \;\; t_{i,max} \geq t_i \geq 0\right\} \tag{1}$$

where $\boldsymbol{J}^T$ is the transpose of the Jacobian matrix consisting of pure-force wrenches along the cables:

$$\boldsymbol{J}^T = [\$_1 \, \ldots \, \$_p] \, , \tag{2}$$

and $\$_i$ is the screw along the i^{th} cable

$$\$_i = \left\{ \begin{array}{c} \vec{u}_i \\ \vec{c}_i \times \vec{u}_i \end{array} \right\}. \tag{3}$$

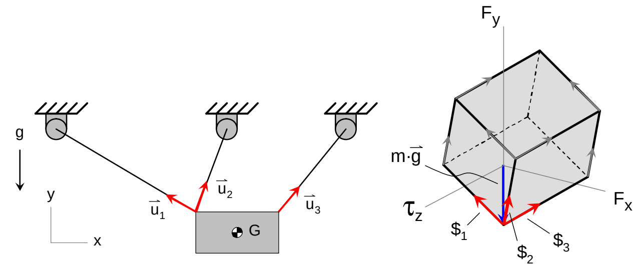
Here $\vec{u}_j$ is the unit vector running along cable i directed away from the end-effector as shown in Figure 2, $\vec{c}_i$ is the vector from G, the center of mass of the end-effector, to the point on the end-effector where cable i is connected and there are p cables attached to the end-effector. The restriction that $t_{i,max} \geq t_i \geq 0$ stems from the fact that each cable can pull but not push (i.e. a cable cannot have negative tension) and is restricted to be less than or equal to a maximum tension $t_{i,max}$. This maximum tension may be determined by the torque limits of the motor reeling in the cable or by the maximum tension a cable can withstand without breaking. Note that the definition in (1) holds for both redundant ($p > n$, where n is the dimension of the task space) and non-redundant ($p \leq n$) manipulators.

We now wish to form the set of wrenches that the end-effector can apply to its surroundings, taking into account the effect of constant external wrenches such as gravity. This set is termed the *Available Net Wrench Set*, abbreviated NW_{avail}. Assuming a constant external wrench $\{\vec{F}^T \vec{\tau}^T\}_{ext}^T$ is present (typically $m\vec{g}$, where m is the combined mass of the end-effector and payload), the applied wrench set can be formed by simply shifting the wrench set in the direction of the external wrench:

$$NW_{avail} = CW \oplus \left\{ \begin{array}{c} \vec{F}_{ext} \\ \vec{\tau}_{ext} \end{array} \right\} \tag{4}$$

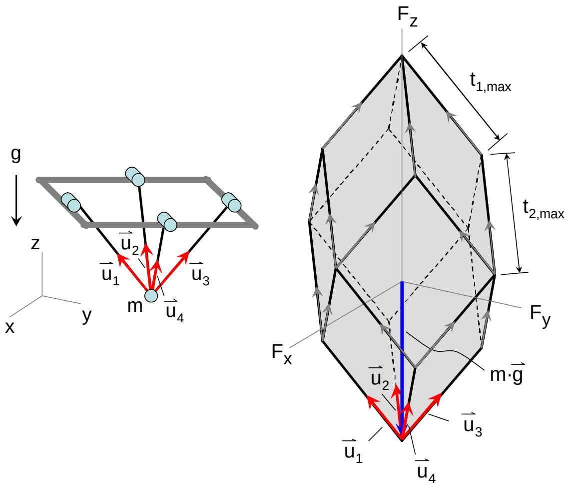
B. Graphical Representation

If the dimension n of the task-space of the robot is less than or equal to three, it is possible to construct a graphical representation of NW_{avail}. As an example, consider the planar manipulator in Figure 3(a). Given the geometry of the manipulator at the current pose, the unit vectors $\vec{u}_1$, $\vec{u}_2$ and $\vec{u}_3$ can be constructed. Applying (3) results in the screws $\$_1$, $\$_2$ and $\$_3$, respectively, which are pure-force wrenches along



(a) Example planar cable robot. (b) Available net wrench set.

Fig. 3. A planar cable robot and its available net wrench set.



(a) Example manipulator. (b) Available net wrench set.

Fig. 4. A 4-cable point-mass cable robot and its available net wrench set.

each of the cables. The set NW_{avail} can then be expressed as $NW_{avail} = \{\$^w : \$^w = a_1 t_{1,max}\$_1 + a_2 t_{2,max}\$_2 + a_3 t_{3,max}\$_3 + m\vec{g};\ 0 < a_i \leq 1\}$. Figure 3(b) illustrates the resulting set where t_{max} is assumed to be the same for all three cables. We can see here that NW_{avail} is a parallelepiped. Note that this parallelepiped is defined in the mixed-dimensional space of F_x-F_y-τ_z.

As a second illustration, consider the manipulator in Figure 4(a). Here the end-effector is a point-mass suspended from four cables. Because the task space has only linear dimensions, the screws $\$_1$ through $\$_4$ are simply $\vec{u}_1$ through $\vec{u}_4$ (the unit vectors along the cables), respectively, resulting in $NW_{avail} = \{\$^w : \$^w = a_1 t_{1,max}\vec{u}_1 + ... + a_4 t_{4,max}\vec{u}_4 + m\vec{g};\ 0 < a_i \leq 1\}$, shown in Figure 4(b). Note that because t_{max} has been assumed to be the same for all four cables and because this set is defined in the force domain only (F_x-F_y-F_z), the length of every edge of NW_{avail} is t_{max}. Also, the geometry of the set is somewhat altered here due to the fact that the number of cables is larger than the degrees of freedom of the task space (i.e. the manipulator is redundant). In this case NW_{avail} is the projection of a four-dimensional hyper-parallelepiped onto three-dimensional space.

In general it can be seen that NW_{avail} is some form of a parallelogram, parallelepiped or hyper-parallelepiped, depending on the number of cables and the dimension of the task-space. In all cases, however, NW_{avail} is a volume bounded by lines (2-D task space), planes (3-D task space) or hyperplanes (task space $>$ 3-D), where the number of boundaries is $2\binom{p}{q-1} = \frac{2p!}{(q-1)!(p-(q-1))!}$, and where p is the number of cables and q is the dimension of the task space.

For manipulators with 2-D or 3-D task spaces the graphical representation of NW_{avail} allows easy visualization of the wrench-generating capabilities of the manipulator at its given pose. In addition, this visualization allows for an intuitive understanding of what causes NW_{avail} to degenerate. When the geometry of the pose of the cable robot causes the Jacobian matrix to become degenerate (its rank decreases), the dimension of the available net wrench set decreases. In parallel robotics, this would be referred to as a singularity pose of the robot. However the unidirectional nature of the cable constraints makes this condition have different meaning for a cable robot and thus the use of the term ‘singularity’ may be somewhat misleading. For example, an underconstrained cable robot may have fewer wires than the dimension of its workspace ($p < q$). Such a robot would have $rank(\boldsymbol{J}) < q$. For a parallel robot this would be considered to be in a singular configuration (which is typically a bad situation), while the same situation for a cable robot may not be problematic and may in fact be preferable, as fewer cables decreases the likelihood of interference. Thus to avoid confusion, the term *wrench deficiency* will be used. A wrench deficiency is a condition where the dimension of the wrench set decreases due to the geometry of the pose. That is, if the maximum rank of $\boldsymbol{J}$ is r_{max}, the robot is in a pose that is wrench deficient if $rank(\boldsymbol{J}) < r_{max}$ for that pose. By constructing the wrench set graphically, it is possible to see geometrically what conditions result in a wrench deficiency.

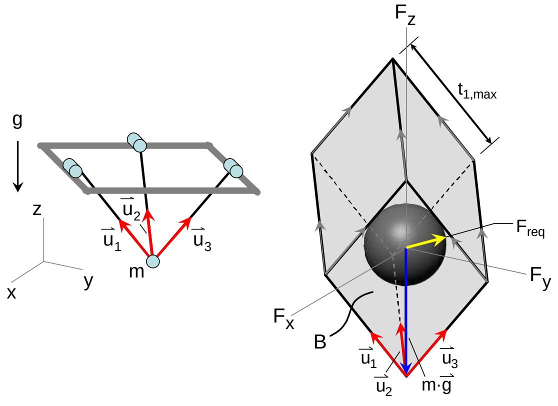
IV. WRENCH-FEASIBLE WORKSPACE

A. *Evaluating Poses*

In many applications, the requirements for a task or set of tasks can be characterized by a required set of wrenches that the end-effector must apply to its surroundings. Given this requirement, the Wrench-Feasible Workspace is defined in [6] as the set of all poses that are *Wrench-Feasible*, i.e. where the manipulator can apply the required set of wrenches. Let this set of required wrenches be called NW_{req}, the *Required Net Wrench Set*. The Wrench-Feasible Workspace can then be described as the set of all poses P of the end-effector where:

$$NW_{req}(P) \subset NW_{avail}(P) \qquad (5)$$

Although NW_{req} can be chosen arbitrarily, it is typically chosen to be a geometrically simple set of wrenches and is independent of P. For example, consider the point-mass 3-cable manipulator shown in Figure 5(a). Common geometries of NW_{req} for point-mass cable robots are circles or squares/rectangles for the planar case and spheres or cylinders



(a) Example manipulator.

(b) Available net wrench set with required net wrench set.

Fig. 5. A point-mass 3-cable manipulator and its available net wrench set containing its required net wrench set.

for the spatial case. Without specific knowledge about the task requirements for the manipulator, it is reasonable to assume the manipulator needs to be able to exert a minimum required force F_{req} in any direction. The corresponding choice for NW_{req} would then be the set of all forces $\vec{F}$ such that $\|\vec{F}\| \leq F_{req}$. Graphically this set NW_{req} is simply a sphere centered at the origin with radius F_{req}. Figure 5 illustrates an example manipulator (5(a)) and its spherical available net wrench set NW_{avail} (5(b)) at that pose. Note that in Figure 5, NW_{req} is completely contained within NW_{avail}; thus this end-effector pose is wrench-feasible and is therefore contained within the Wrench-Feasible Workspace of this manipulator.

This geometric construction of NW_{req} not only allows wrench-feasibility to be visualized, but also allows wrench-feasibility to be determined by simple geometric calculation. In this case, determining analytically if the pose is wrench-feasible reduces to simply testing whether the distances between the planes that define the boundaries of NW_{avail} and the origin are greater than or equal to F_{req}. In general, it is very likely that the geometry of NW_{req} will be simple, resulting in simple geometric conditions for $NW_{req} \subset NW_{avail}$. In the case that NW_{req} is more complex, an approximation of this set can be made by choosing a simpler geometry that contains NW_{req}. Using this approximate set will give a conservative estimate of whether or not the pose is wrench-feasible.

While visualization of these sets may break down for cases where the dimension of the taskspace is higher than three, the geometric conditions for wrench-feasibility do not. Thus for higher-dimensional task spaces the wrench-feasibility of a pose can be calculated from the distance between the hyperplanes that bound NW_{avail} and the hyper-object (such as a hypersphere) that represents NW_{req}.

B. Constructing the Wrench-Feasible Workspace

While it is very advantageous to now be able to check whether or not a pose is wrench-feasible by simple geometric conditions, it is still necessary to construct the entire wrench-feasible workspace. A simple method to accomplish this would be to discretize the task space and evaluate each discrete pose of the manipulator, thus constructing a discretized wrench-feasible workspace as the set of discrete poses that are wrench-feasible. Such a method, while relatively simple, would be time-consuming, computationally expensive and would not produce a complete description of the continuous wrench-feasible workspace.

Instead, the wrench-feasible workspace can be constructed by generating the boundaries of the workspace analytically. The boundaries of this workspace consist of the set of all poses of the manipulator such that $NW_{req} \subset NW_{avail}$ and one or more of the planes bounding NW_{avail} contact NW_{req}. The conditions for this to occur can be represented as geometric conditions on the geometry of the pose. Each plane defining a boundary of NW_{avail} can be expressed as a function of the screws $\$_1$ through $\$_p$. The condition of contact between one of these planes and NW_{req} results in a relationship between the screws $\$_1$ through $\$_p$ that causes contact to occur. Note that $\$_1, ..., \$_p$ change with P. This relationship can then be used to construct the geometric conditions that result in the pose being on the boundary of the workspace. Thus these geometric relationships represent an analytical definition of a boundary of the force feasible workspace. Repeating this process for each of the planes that bounds NW_{avail} results in a set of analytical inequalities that define the force feasible workspace. Note that because there are $2\binom{p}{q-1}$ sides that bound NW_{avail}, there are $2\binom{p}{q-1}$ workspace boundaries that must be formulated.

As a simple example, consider again the manipulator in Figure 5(a). Because the end-effector is a point mass, the screws $\$_1$, $\$_2$ and $\$_3$ are simply $\vec{u}_1$, $\vec{u}_2$ and $\vec{u}_3$, respectively. Let us construct the workspace boundary associated with contact between NW_{req} and the lower front boundary of NW_{avail} (labeled as B in Figure 5(b)). The plane that defines this boundary is $B(x, y, z) = a\vec{u}_1 + b\vec{u}_3 + m\vec{g}$ where $a, b \in I\!R$. The condition for contact between this plane and NW_{req} is simply that the perpendicular distance between $B(x, y, z)$ and the origin is F_{req}. As discussed in companion paper [14], this results in a boundary of the wrench-feasible workspace being a plane that passes through the locations of motors 1 and 2 with an angle of $\theta = \cos^{-1}\frac{F_{req}}{\|m\vec{g}\|}$, where $0 \leq \theta \leq \frac{\pi}{2}$.

C. Discussion

The procedure outlined here results in analytical representations of the boundaries of the wrench-feasible workspace. However, forming explicit representations of the boundaries may not be easy. In some cases, the boundaries may need to be left in implicit formulations and constructed numerically by discretizing the boundaries. In addition, this procedure becomes more challenging for manipulators with higher-dimensional taskspaces and for applications that have complicated geometries for NW_{req}. For a more detailed discussion of implementing this method for point-mass cable robots see [14].

V. CONSTRUCTING OTHER WORKSPACES

One of the additional benefits of this analysis approach is that several previously proposed workspaces can be described using this theoretical framework. The statically reachable workspace, defined in [7] as the set of all end-effector poses that can be reached statically, is actually a special case of the wrench-feasible workspace where $NW_{req} = \{\vec{0}\}$. The controllable workspace, defined in [8], is a special case of the wrench-feasible workspace where NW_{req} is a single point in the wrench space. Thus the method presented here for analytically forming the wrench-feasible workspace can be used to analytically form the statically reachable workspace and controllable workspace.

The dynamic workspace, defined in [5] as the set of all poses of the end-effector where the end-effector can be given a specific acceleration, can also be formed using the method presented here. Assuming all cables are in tension, for a given pose and instantaneous velocities and accelerations the dynamic equations of motion are as follows:

$$\boldsymbol{M}\left\{\begin{array}{c}\ddot{\vec{x}}\\ \dot{\vec{\omega}}\end{array}\right\} = \boldsymbol{J}^T\left\{\vec{t}\right\} + \left\{\begin{array}{c}\vec{F}_{ext}\\ \vec{\tau}_{ext}\end{array}\right\} + \vec{B}\left(\dot{\vec{x}},\vec{\omega}\right) \tag{6}$$

where $\boldsymbol{M}$ is the inertia matrix of the end-effector defined about G, $\{\vec{F}_{ext}^T, \vec{\tau}_{ext}^T\}^T$ is the external wrench applied at the center of gravity of the end-effector, and $\vec{B}$ contains all other dynamic effects (i.e. gyroscopic effects, damping, etc.). Note that it is assumed that the inertial effects of the cables and the motors is very small compared to the inertial effects of the end-effector and thus can be neglected. The set of all possible accelerations that the end-effector can be given without violating the tension limits in the cables can then be defined as A_{avail}, the *Available Acceleration Set*, where:

$$A_{avail} = \boldsymbol{M}^{-1}\left[NW_{avail} \oplus \vec{B}\left(\dot{\vec{x}},\vec{\omega}\right)\right]. \tag{7}$$

The set A_{avail} is simply NW_{avail} shifted by $\vec{B}$ and scaled by $\boldsymbol{M}^{-1}$. Thus A_{avail}, like NW_{avail}, is some form of a parallelogram, parallelepiped or hyper-parallelepiped. If we define the set of accelerations that are required of the end-effector at this pose as A_{req}, the *Required Acceleration Set*, the dynamic workspace of the manipulator can be defined as the set of poses P such that:

$$A_{req}(P) \subset A_{avail}(P). \tag{8}$$

Because the geometric properties of A_{avail} are the same as NW_{avail}, the same geometric techniques can be used to form the dynamic workspace analytically. In fact, this construction allows a more general definition of the dynamic workspace. The dynamic workspace as defined by Barrette et al. would be limited to $A_{req}(P)$ consisting of a single acceleration of the end-effector. Here we can extend this definition to allow $A_{req}(P)$ to consist of a *set* of accelerations.

VI. OTHER APPLICATIONS

A. Optimal Control Using Acceleration Limits

The Available Acceleration Set (A_{avail}) may also be incorporated into a control scheme for the cable robot. The set A_{avail} establishes limits on what accelerations should be demanded of the end-effector at pose P. Using A_{avail} to bound the accelerations of the end-effector, the method proposed by Bobrow et al. [15] may be applied to determine the time-optimal trajectory of the manipulator along a desired path. By keeping the acceleration of the end-effector within A_{avail}, it is guaranteed that a trajectory can be followed without wires going slack, wires exceeding their maximum tension, or the end-effector leaving its desired trajectory. In addition, if a payload is suspended from the end-effector it may be desirable to apply input-shaping to the resulting trajectory in order to prevent oscillation of the payload as described in [16].

B. Payload Specification

Given $NW_{avail}(P)$ and $NW_{req}(P)$, it is possible to use these sets to specify the maximum payload of the cable robot. If the manipulator has a wrench-feasible workspace WFW and is required to operate in a region T within the task space, where $T \subset WFW$, then at any pose in T the maximum payload can be determined by adding mass to the end-effector until NW_{req} is no longer completely contained within NW_{avail}. The highest load at which NW_{req} is still contained within NW_{avail} is the maximum payload for that pose. The maximum payload for each pose can be found and the payload for the manipulator can be specified as the smallest of these payloads. That is, if the mass of the end-effector is m and each pose P has a maximum possible load $L(P)$ that can be added to m while keeping NW_{req} inside the NW_{avail} (i.e. maintaining force feasibility), then the maximum payload of the manipulator can be specified as:

$$Payload = \min_{P \in T} L(P) \tag{9}$$

C. Analysis of Failure Modes

If a cable robot is required to exert wrenches that exceed its capabilities, the robot will fail. The available net wrench set can be used in order to analyze the type of failure that will occur. Given NW_{avail}, this set can be expanded by removing the upper bound on the wire tensions ($t_{max} \rightarrow \infty$). The space of all wrenches can then be divided into three regions, each representing a different behavior of the manipulator.

This is illustrated in Figure 6 for the manipulator in Figure 5. The shaded region represents the set of all wrenches that can be resisted if $t_{max} \rightarrow \infty$. The space of all wrenches can then be subdivided into three regions: NW_{avail}, R_1 and R_2. Note that R_1 includes all of the shaded region *except* the region contained within NW_{avail}. If the required wrench lies in NW_{avail}, the manipulator can apply the required wrench. If the wrench lies in R_1 or R_2 the manipulator fails to apply the wrench, but in two different ways. If the wrench lies in R_1 the manipulator fails because tension limits have been exceeded, which can be shown using the Jacobian relationship in (1). This may result in damage to the cables, motors or end-effector. If, however, the wrench lies in R_2, the manipulator fails because one or more cables goes slack because solving for

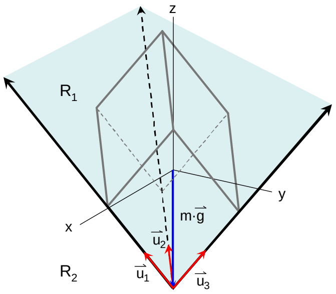


Fig. 6. Diagram of Failure Regions using a Resisted Wrench Set.

the tensions using (1) results in one or more negative tensions. This will cause the pose of the end-effector to change. Also, depending on the magnitude of the wrench, the tension limits may be exceeded in the cables that are taut.

VII. DISCUSSION

The key point of the analysis approach presented here is that it is much easier to determine if a pose is wrench-feasible by performing the analysis in the task space, while traditionally much of the analysis was performed in the joint space. In a way this is similar to the configuration space approach in mobile robots [17], where objects are mapped to configuration space and then a desired configuration is tested to be in the object-free set.

There are some difficulties with this approach that need to be addressed. In order to perform this analysis the pose of the end-effector must be known. If only the cable lengths are known it is not trivial to find the resulting pose of the end-effector, particularly for underconstrained cable robots. The analysis would also need to be modified if the end-effector is not a single rigid body.

VIII. CONCLUSION AND FUTURE WORK

Despite the many potential uses for cable robots, relatively few analytical tools are available for the design and optimization of cable robots. This paper has begun to address this deficiency by proposing an analysis approach based on wrenches. The available net wrench set was introduced as a description of the wrench-generating capability of the manipulator at a pose. In many cases this set can be constructed graphically, providing a way to visualize the capabilities of the manipulator as well as providing a geometric understanding of the situations that result in wrench deficiencies. Because the available net wrench set is bounded by lines or planes, the wrench-feasibility of the pose can be calculated by simple geometric relationships. A geometric method of calculating the wrench-feasible workspace was presented that took advantage of the geometric structure of the available net wrench set. It was then shown that this method could be extended to include calculation of the statically reachable workspace, controllable workspace and dynamic workspace. In fact, this approach allows generalization of the definition of the dynamic workspace. In addition, several other applications were proposed that are based on the available net wrench set, including optimal control, payload specification and failure analysis.

There is a great deal of future work that can be done in this area. Analytical definitions of wrench-feasible workspaces must be defined not only for the case of simple spherical NW_{req}s, but also for more general geometries, such as polyhedra. While a number of analysis tools have been presented in this paper, many more could potentially be developed using the available net wrench set as a basis. In addition, it may be of interest to incorporate the effects of cable sag and stretch.

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