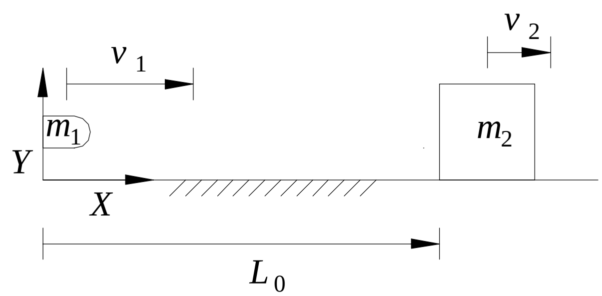
Conservation of Linear Momentum: Bullet striking Box

A bullet of mass m_1 is fired with a constant velocity v_1 into a point mass m_2 traveling at constant initial velocity v_2, as shown in the diagram below. We assume both masses are particles, there is no wind resistance for the bullet, and there is no friction between the motion surface and second point mass. We further assume that the bullet is imbedded into the second mass at collision. Under these conditions, the principle of Conservation of Linear Momentum applies.



Conservation of Linear Momentum

When the sum of external impulses acting on a system of particles is zero, the linear momenta for this system of particles remains constant during time period t_A to t_B. Mathematically, this principle may be expressed as:

$$\sum \left(m_i \mathbf{v}_i\right)_A = \sum \left(m_i \mathbf{v}_i\right)_B$$

where m_i is the mass of point mass i, $\mathbf{v}_i$ is the velocity vector of point mass i, time period t_A is indicated by the left-hand side of the above equation, and time period t_B is indicated by the right-hand side. The summation is for $i = 1, 2, \cdots, n$ particles.

For the bullet/particle problem with the assumptions proposed above, Conservation of Linear Momentum yields the following equation (both velocity vectors are in the horizontal X direction):

$$m_1 v_1 + m_2 v_2 = \left(m_1 + m_2\right) v_f$$

Time period A is before impact and time period B is after impact, when the two masses have joined and are traveling as one. The final velocity (in the X direction) is:

$$v_f = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$$

Note although the velocity vectors are constrained to be along the X direction in this problem, their magnitudes can be positive or negative; some combinations will not yield impact. The kinematics equations are simple; the following equations apply to mass 1 or mass 2 before impact, and the combination of mass 1 and mass 2 after impact: substitute v_1, v_2, v_f for v, as desired; also include the appropriate initial condition on X position, x_0:

$$a_x(t) = 0$$
$$v_x(t) = v$$
$$x(t) = x_0 + vt$$

The time to collision can be calculated by setting the two displacements equal:

$$x_1 = x_2$$
$$x_{01} + v_1 t_{coll} = x_{02} + v_2 t_{coll}$$

Therefore: $$t_{coll} = \frac{x_{02} - x_{01}}{v_1 - v_2}$$

We can always set the initial bullet displacement to zero by shifting the X axis origin to the bullet departure point. Therefore $x_{01} = 0$ and $x_{02} - x_{01} = L_0 - 0 = L_0$ in the equations above, where L_0 is shown in the system figure above.

User sets: v_1 and v_2

$$0 \le v_1 \le 1000$$
$$-50 \le v_2 \le 50$$

Computer sets: $m_1 = 0.10$ *kg* and $m_2 = 10$ *kg*; also initial displacements $x_{01} = 0$ and $x_{02} = 100$ *m*.

Visualize: motion before and after bullet impact, plus kinematics plots for v, x.

Numerical Display: v_f, t_{coll}, initial momentum 1 $m_1 v_1$, initial momentum 2 $m_2 v_2$, and total momentum $m_1 v_1 + m_2 v_2 = (m_1 + m_2) v_f$.

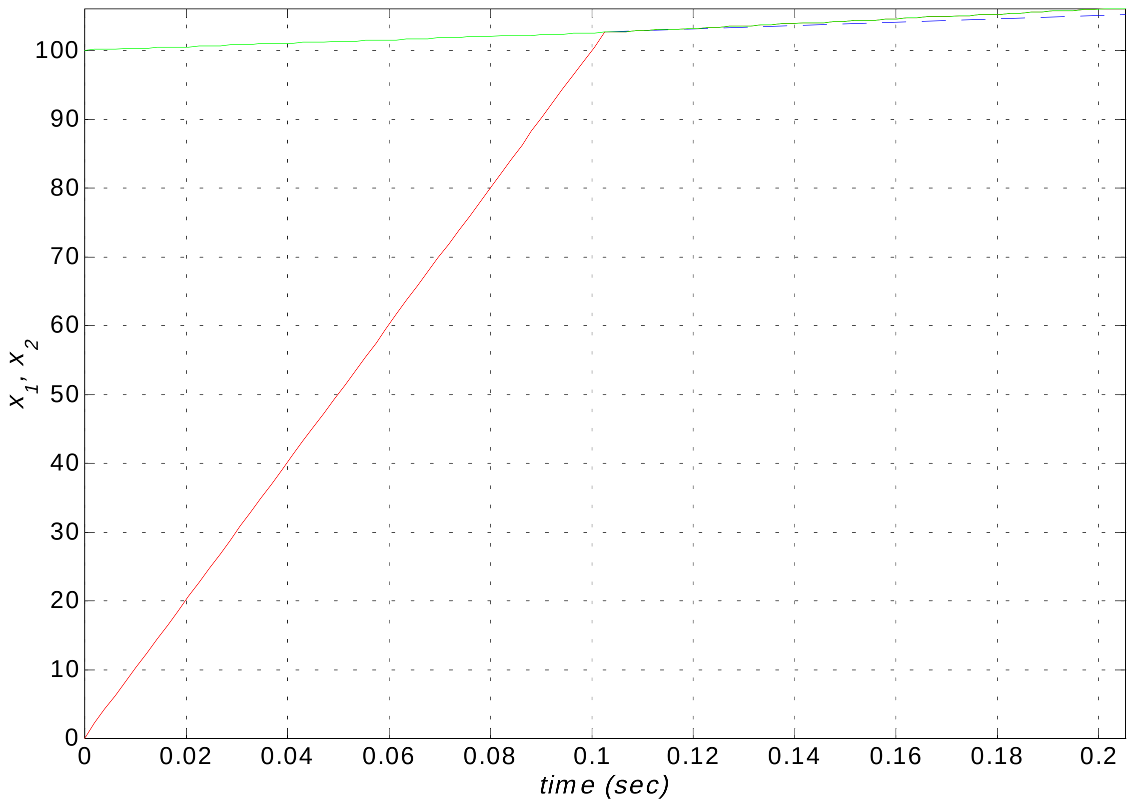
User Feels: momentum $m_1 v_1$, $m_2 v_2$, or $(m_1 + m_2) v_f$ (user chooses).

Example: When the user enters $v_1 = 1000$ *m/s* and $v_2 = 25$ *m/s*, the numerical results are:

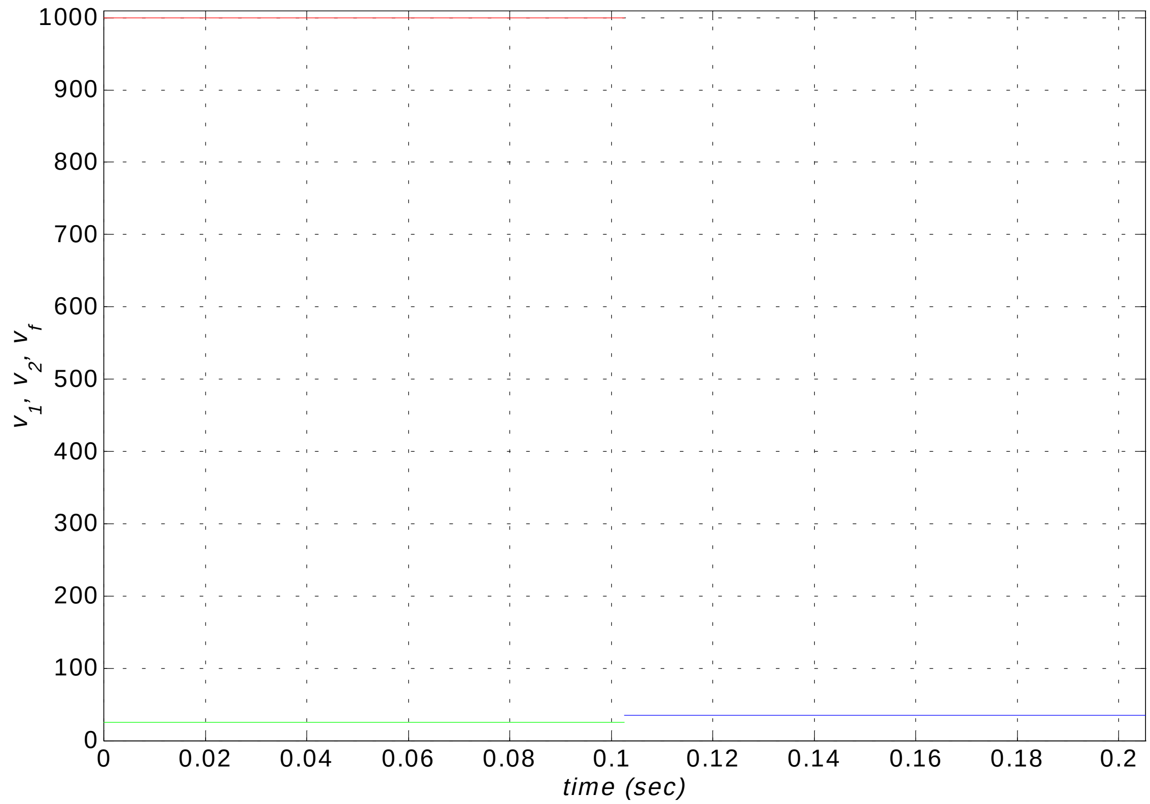
$v_f = 34.65$ *m/s* and $t_{coll} = 0.1026$ *s*;

$m_1 v_1 = 100$ *kgm/s*, $m_2 v_2 = 250$ *kgm/s*, and $m_1 v_1 + m_2 v_2 = (m_1 + m_2) v_f = 350$ *kgm/s*

For this example, the associated kinematics plots vs. time are shown below. After t_{coll}, the two masses move as one with constant final velocity v_f; the blue dashed line, an extension of the x_2 plot, is shown to emphasize the increase in slope ($v_f > v_2$), as shown in the velocity plot. In the first plot below, the bullet displacement starts from $x_{01} = 0$ *m* and the point mass 2 displacement starts from $x_{02} = 100$ *m*. In the second plot below, the bullet velocity is high and the point mass 2 velocity is relatively lower; after impact at t_{coll}, $(m_1 + m_2)$ has the constant velocity v_f, only slightly higher than v_2.



Conservation of Linear Momentum: Point Mass Displacements



Conservation of Linear Momentum: Point Mass Velocities

Comprehension Assignment:

Once you get the 'feel' for this simulation, run the program several times to collect and plot data: for a constant bullet velocity of v_1 = 1000 m/s to the right, vary the point mass velocity v_2 over its allowable $\pm$ range and determine the resulting final velocity v_f, collision time t_{coll}, initial momentum 1 $m_1 v_1$, initial momentum 2 $m_2 v_2$, and total momentum $m_1 v_1 + m_2 v_2 = (m_1 + m_2) v_f$. Plot v_f, t_{coll}, $m_1 v_1$, $m_2 v_2$, and $(m_1 + m_2) v_f$. vs. v_2. Discuss the trends you see – do the results make sense physically?