

ChE 400: Differential Equations: Analytical Solutions using Laplace Transform (H-7b)

Gerardine G. Botte

Objectives

- **General:**
 - Apply the concepts of this chapter for the solution of chemical engineering problems that require the calculation of homogeneous/no homogeneous-linear-ordinary differential equations (single equations and systems of equations)
- **Specific objectives:**
 - Calculate the Laplace transform of a function using the tables and properties
 - Calculate the inverse Laplace transform using the partial fraction method
 - Calculate Laplace transform and inverse of Laplace transform using Matlab
 - Recognize the type of differential equations that can be solved using the Laplace Transform method
 - Solve ODEs using the Laplace transform method
 - Solve systems of ODEs using the Laplace transform method

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Outline

- Laplace Transform
 - Definition
 - Conditions for its existence
 - Tables
 - Properties
- Inverse Laplace Transform
 - Definition
 - Properties
- Laplace Transform to solve ODE
 - Single ODE
 - Systems of ODEs
- ODE Modeling Exercise
 - Use of Matlab

- Laplace Transform
 - Definition
 - Conditions for its existence
 - Laplace Transform Tables
 - Properties
 - Inverse Laplace Transform
 - Definition
 - Properties
 - Laplace Transform to solve ODE
 - Single ODE
 - Systems of ODEs
 - ODE Modeling Exercise
 - Lab Practice
 - Use of Matlab to calculate Laplace transform
- Tutorial 5 will be very useful for your project!!!
- Tutorial 5 ← Tutorial 5

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<p>Laplace Transform</p> <ul style="list-style-type: none"> - Definition - Conditions for its existence - Tables - Properties <p>Inverse Laplace Transform</p> <ul style="list-style-type: none"> - Definition - Properties <p>Laplace Transform to solve ODE</p> <ul style="list-style-type: none"> - Single ODE - Systems of ODEs <p>ODE Modeling Exercise</p> <p>Lab Practice</p> <ul style="list-style-type: none"> - Use of Matlab 	<h2 style="margin: 0;">Laplace Transform</h2> <hr/> <ul style="list-style-type: none"> • Let $f(x)$ be defined for $0 \leq x < \infty$ and let “s” denote an arbitrary real variable. The Laplace transform of $f(x)$, designated by either $\mathcal{L}\{f(x)\}$ or $F(s)$, is $\mathcal{L}\{f(x)\} = F(s) = \int_0^{\infty} e^{-sx} f(x) dx \quad (1)$ <ul style="list-style-type: none"> • For all the values of “s” for which the improper integral <u>converges</u>. When evaluating the integral the variable “s” is treated as a constant, since the integration is with respect to “x” <p style="text-align: right; font-size: small;">CHE 400 10/4/2005 4</p>
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<p>Laplace Transform</p> <ul style="list-style-type: none"> - Definition - Conditions for its existence - Tables - Properties <p>Inverse Laplace Transform</p> <ul style="list-style-type: none"> - Definition - Properties <p>Laplace Transform to solve ODE</p> <ul style="list-style-type: none"> - Single ODE - Systems of ODEs <p>ODE Modeling Exercise</p> <p>Lab Practice</p> <ul style="list-style-type: none"> - Use of Matlab 	<h2 style="margin: 0;">Improper Integral Solution</h2> <hr/> <ul style="list-style-type: none"> • The improper integral is defined by $\int_a^{\infty} g(x) dx = \lim_{R \rightarrow \infty} \int_a^R g(x) dx \quad (2)$ <ul style="list-style-type: none"> • If the limit exists. When the limit exists the improper integral is said to <u>converge</u>, otherwise the improper integral is said to diverge <p style="text-align: right; font-size: small;">CHE 400 10/4/2005 5</p>
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<p>Laplace Transform</p> <ul style="list-style-type: none"> - Definition - Conditions for its existence - Tables - Properties <p>Inverse Laplace Transform</p> <ul style="list-style-type: none"> - Definition - Properties <p>Laplace Transform to solve ODE</p> <ul style="list-style-type: none"> - Single ODE - Systems of ODEs <p>ODE Modeling Exercise</p> <p>Lab Practice</p> <ul style="list-style-type: none"> - Use of Matlab 	<h2 style="margin: 0;">Conditions that guarantee Existence of Laplace Transform</h2> <hr/> <ul style="list-style-type: none"> • Not all functions have a Laplace transform • The Laplace transform only exists if the integral converges • In this course, we will work with functions for which the Laplace transform exists • For more information on convergence criteria for improper integrals read chapter 14 of Schaum's differential Eqs. Book (see recommended reading slide). <p style="text-align: right; font-size: small;">CHE 400 10/4/2005 6</p>
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Example 1

- Calculate the Laplace transform of $f(x)=1$

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Laplace transform Tables

Laplace Transform

- Definition
- Conditions for its existence
- Tables
- Properties

Inverse Laplace Transform

- Definition
- Properties

Laplace Transform to solve ODE

- Single ODE
- Systems of ODEs

ODE Modeling Exercise

- Lab Practice
- Use of Matlab

- The Laplace transform of a function is calculated using the definition (Eq. 1) as well as different properties
- The Laplace transforms of different functions have been tabulated.
- In this course we will use the Laplace transform tables combined with the Laplace transform properties to determine the Laplace transforms of functions as well as the inverse Laplace transforms

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Exercise 2

- Determine the Laplace transform of the following functions, using the Laplace transform table provided in class

$$f(x) = x \quad (a) \quad f(x) = x \sin ax \quad (b)$$

$$f(x) = \sin(ax + b) \quad (c) \quad f(x) = \frac{1 - e^{ax}}{x} \quad (d)$$

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Laplace Transform - Definition - Conditions for its existence - Tables - Properties Inverse Laplace Transform - Definition - Properties Laplace Transform to solve ODE - Single ODE - Systems of ODEs ODE Modeling Exercise Lab Practice - Use of Matlab	<h2 style="margin: 0;">Properties <u>Linearity</u></h2> <hr/>
	<ul style="list-style-type: none"> • If “a” and “b” are arbitrary constants: $\mathcal{L}[af(x) + bg(x)] = aF(s) + bG(s)$ <p style="text-align: right; margin-top: 20px;"> <small>ChE 400 10/4/2005 10</small> </p>

<h2 style="margin: 0;">Exercise 3</h2> <hr/>
<ul style="list-style-type: none"> • Find F(s) if $f(x) = 2\sin(x) + 3\cos(2x)$ <p style="text-align: right; margin-top: 20px;"> <small>ChE 400 10/4/2005 11</small> </p>

Laplace Transform - Definition - Conditions for its existence - Tables - Properties Inverse Laplace Transform - Definition - Properties Laplace Transform to solve ODE - Single ODE - Systems of ODEs ODE Modeling Exercise Lab Practice - Use of Matlab	<h2 style="margin: 0;">Properties <u>First Shift Theorem</u></h2> <hr/>
	<ul style="list-style-type: none"> • For any constant “a”: $\mathcal{L}[e^{ax}f(x)] = F(s - a)$ <p style="text-align: right; margin-top: 20px;"> <small>ChE 400 10/4/2005 12</small> </p>

Exercise 4

- Find $\mathcal{L}[xe^{4x}f(x)]$

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Properties Product of X by function

Laplace Transform

- Definition
- Conditions for its existence
- Tables
- Properties

Inverse Laplace Transform

- Definition
- Properties

Laplace Transform to solve ODE

- Single ODE
- Systems of ODEs

ODE Modeling Exercise

- Lab Practice
- Use of Matlab

- For any positive integer “n”:

$$\mathcal{L}[x^n f(x)] = (-1)^n \frac{d^n}{ds^n} [F(s)]$$

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Exercise 5

- Find $\mathcal{L}[x^{7/2}]$

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Laplace Transform - Definition - Conditions for its existence - Tables - Properties Inverse Laplace Transform - Definition - Properties Laplace Transform to solve ODE - Single ODE - Systems of ODEs ODE Modeling Exercise Lab Practice - Use of Matlab	<h2>Properties <u>Linearity</u></h2> <hr/>
	<ul style="list-style-type: none"> • If “a” and “b” are arbitrary constants: $\mathcal{L}^{-1}[aF(s) + bG(s)] = a\mathcal{L}^{-1}[F(s)] + b\mathcal{L}^{-1}[G(s)]$
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Laplace Transform - Definition - Conditions for its existence - Tables - Properties Inverse Laplace Transform - Definition - Properties Laplace Transform to solve ODE - Single ODE - Systems of ODEs ODE Modeling Exercise Lab Practice - Use of Matlab	<h2>Properties <u>First Shift Theorem</u></h2> <hr/>
	<ul style="list-style-type: none"> • For any constant “a”: $\mathcal{L}^{-1}[F(s - a)] = e^{at} \mathcal{L}^{-1}[F(s)]$
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Laplace Transform - Definition - Conditions for its existence - Tables - Properties Inverse Laplace Transform - Definition - Properties Laplace Transform to solve ODE - Single ODE - Systems of ODEs ODE Modeling Exercise Lab Practice - Use of Matlab	<h2>Finding Inverse Laplace Transform</h2> <hr/>
	<ul style="list-style-type: none"> • Finding the inverse Laplace transform is a very crucial step to solve ODE • We need to use the Laplace transform tables along with the properties of inverse Laplace transform to determine the function f(x) • Most of the time we end up finding the inverse Laplace transform of functions which are ratios of polynomials. We will use Matlab to find the inverse of the Laplace Transform in this course
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Exercise 6

- Find the inverse Laplace transform of the function using Matlab (see Tutorial V):

$$F(s) = \frac{8}{s^3(s^2 - s - 2)}$$

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Exercise 7

- Find the inverse Laplace transform of the function (see Tutorial V):

$$F(s) = \frac{1}{(s^2 + 1)(s^2 + 4s + 8)}$$

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Solution of Single ODE

Laplace Transform
- Definition
- Conditions for its existence
- Tables
- Properties

Inverse Laplace Transform
- Definition
- Properties
- Partial Fraction Method

Laplace Transform to solve ODE
- Single ODE
- Systems of ODEs

ODE Modeling Exercise

Lab Practice
- Use of Matlab

- The method only works for ODE with constant coefficients
- The equations can be homogeneous and no homogeneous

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Laplace Transform - Definition - Conditions for its existence - Tables - Properties Inverse Laplace Transform - Definition - Properties - Partial Fraction Method Laplace Transform to solve ODE - Single ODE - Systems of ODEs ODE Modeling Exercise Lab Practice - Use of Matlab	<h2>Solution of Single ODE Procedure</h2> <hr/> <ol style="list-style-type: none"> 1. Apply Laplace transform to the ODE, (need to use derivative property: Eq. (3), Eqs. 4 and 5) 2. Apply boundary conditions 3. Solve for F(s) 4. Get Inverse Laplace transform of F(s)
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<h2>Exercise 8</h2> <hr/> <ul style="list-style-type: none"> • Solve the following differential equation: $y' - 5y = e^{5x}$ <p>at $x=0, y = 0$</p>
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Laplace Transform - Definition - Conditions for its existence - Tables - Properties Inverse Laplace Transform - Definition - Properties - Partial Fraction Method Laplace Transform to solve ODE - Single ODE - Systems of ODEs ODE Modeling Exercise Lab Practice - Use of Matlab	<h2>Solution of Systems of ODE Procedure</h2> <hr/> <ol style="list-style-type: none"> 1. Apply Laplace transform to the system of ODE, (need to use derivative property: Eq. (3), Eqs. 4 and 5) 2. Apply boundary conditions 3. Solve for F(s) 4. Get Inverse Laplace transform of F(s)
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Exercise 9

- Solve the following Equations

$$u' + u - v = 0$$

$$v' - u + v = 2$$

$$\text{at } x=0, u = 1, v = 2$$

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Modeling Exercise

Laplace Transform

- Definition
- Conditions for its existence
- Tables
- Properties

Inverse Laplace Transform

- Definition
- Properties
- Partial Fraction Method

Laplace Transform to solve ODE

- Single ODE
- Systems of ODEs

ODE Modeling Exercise

- Lab Practice
- Use of Matlab

- Solve the modeling Exercise (tutorial 5) posted on the web:
<http://www.ent.ohiou.edu/che/che400/Handouts%20and%20Class%20Notes.htm>

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Matlab Practice

Calculation of Laplace Transform and Inverse Laplace Transform

Laplace Transform

- Definition
- Conditions for its existence
- Tables
- Properties

Inverse Laplace Transform

- Definition
- Properties
- Partial Fraction Method

Laplace Transform to solve ODE

- Single ODE
- Systems of ODEs

ODE Modeling Exercise

- Lab Practice
- Use of Matlab

- Reproduce the exercises given in Tutorial 5: Use of Matlab to Calculate Laplace Transforms, posted on the web:
<http://www.ent.ohiou.edu/che/che400/Handouts%20and%20Class%20Notes.htm>

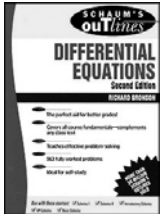
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Recommended Reading and Assignment

- Read chapters 14, and 15 of Schaum's differential equations book
- Read chapters 17, and 18 (and solve the problems given in these chapters) of Schaum's differential equations book



QA372 .B856 1994eb
ELECTRONIC BOOK
Author Bronson, Richard
Title **Schaum's outline of theory and problems of differential equations [computer file] / Richard Bronson**
Edition 2nd ed
Imprint New York : McGraw-Hill, c1994
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Summary

- Do all the exercises done in class by yourself
- Do recommended assignment
- You must be able to calculate Laplace transforms
- You must be able to decompose a fraction into partial fractions
- You must be able to calculate an inverse Laplace transform
- You must be able to solve linear ODE with constant coefficients (using Laplace Transform Method)
 - Single equations
 - Systems of equations

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